Downhole Pipe Forces Model

Prepared for IMPACT Technologies LLC

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Introduction

A downhole forces modeling was requested by IMPACT Technologies LLC. This study developed a model to predict the shape of the pipe at downhole conditions and determines if the various pipe string sizes can reach a specified landing depth. The wellbore trajectory will be described by this study in a plot with depth in feet and Inclination in degrees.

Analysis Objective

The primary objective of this modeling is to use the model developed to calculate:

1. Horizontal displacement and vertical displacement of the pipe tip relative to the support point
2. Angle off horizontal at the pipe tip
3. Maximum allowed pipe length to avoid the pipe yield at the support point

Sensitivity analysis, including temperature effect, will be performed.

4. Calculate when pipe contacts bottom of cut hole for a new support point

Modeling Data

The customer provided the following data (Table 1).

Table 1 – Customer Data	
pipe sizes	1/2", 3/4", 1", 1-1/4", 1-1/2"
Materials	Stainless steel, Chrome 13, Standard steel
Internal fluid	7 ～ 12 lbs/gal
External fluid	0 ～ 12 lbs/gal
Pipe angle at support point	0 to 90 degrees off horizontal
Downhole temperature	Up to 500 °F

The following material property data, shown in Table 2, are used in this study.

Table 2 – Material Properties			
	Density lbs/in^3	**Young's Modulus** kpsi	**Yield Strength** kpsi
Stainless Steel	0.29	30,000	40
Chrome 13	0.284	30,000	80
ASTM A36	0.283	30,000	36
ASTM A572	0.283	30,000	50
ASTM A 514	0.283	30,000	100

Both Young's modulus and yield strength of steels will decrease with increased downhole temperature. This study used the following empirical formula for Young's modulus at various temperatures, based on ASME B31.1 – 1995,

$$E = E_{70} - 5.833 \times (T - 70)$$

where E_{70} is the Young's modulus at 70 °F (in kpsi) and T is the temperature in °F.

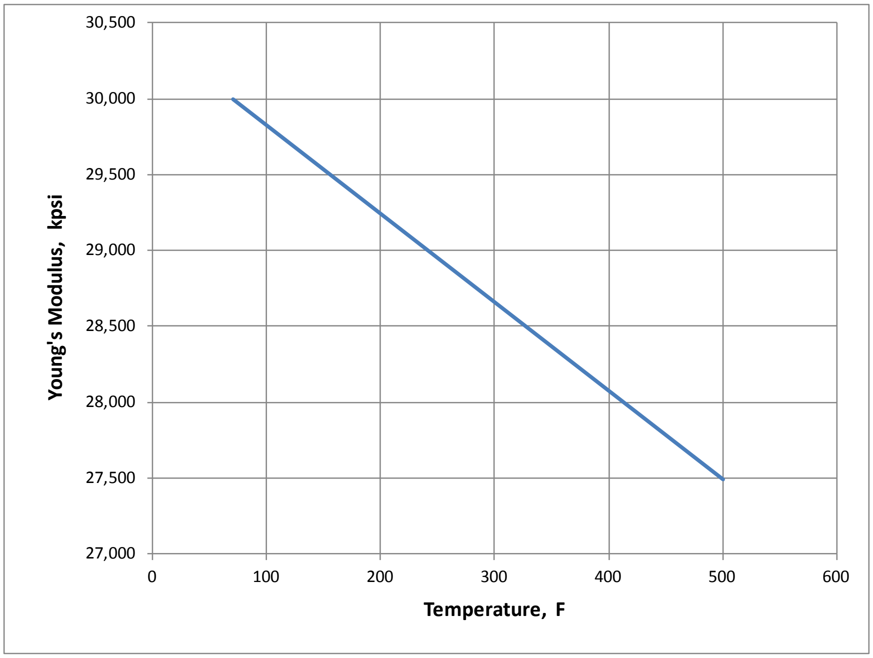


Figure 1 – Effect of Temperature on Young's Modulus

Also an empirical formula was used to include the effect of temperature on pipe yield strength. The validity range for temperature is between 122 °F and 550 °F.

$$YS = YS_0 \times \left[1 - \frac{1}{4500}(T - 122)\right]$$

where YS_0 is the yield strength at room temperature.

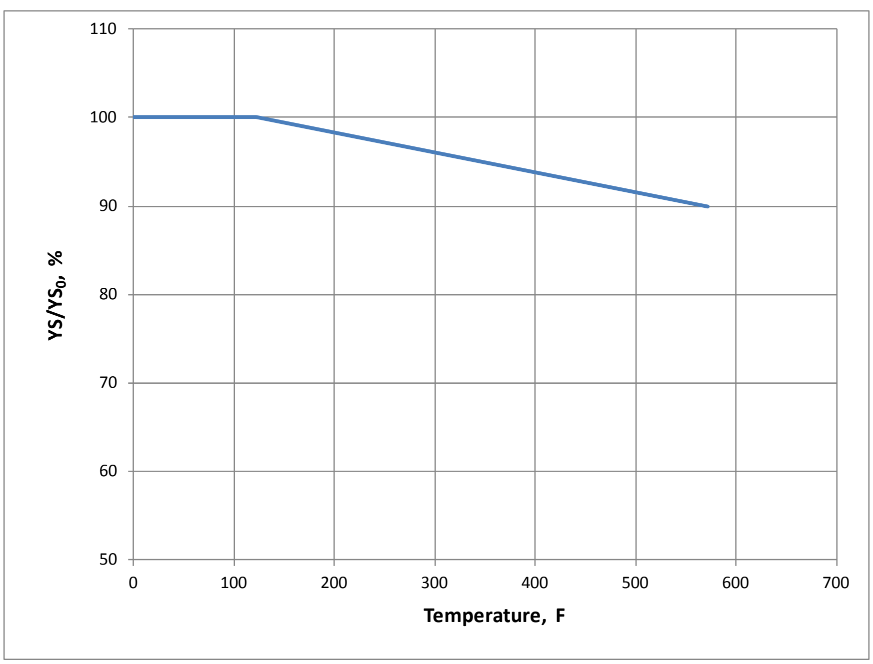


Figure 2 – Effect of Temperature on Yield Strength

Numerical Analysis Model

A Large Deflection Beam Model

A cantilever beam model has been developed to meet the needs of calculating the pipe deflection at downhole. It is assumed that the pipe is rigidly fixed at a support point with a specified inclination angle and that, below the support point, the tip of the pipe is allowed to hang freely in an “open tank”. There are fluids inside the pipe and the “open tank”, respectively. The fluids can be different. The only load is the buoyed weight of the pipe based on the pipe weight and the fluids densities. Since this study needs calculate the maximum allowed pipe length between the support point and the tip, a large deflection model must be utilized.

The details of the model are described in the Appendix A. For the purpose of verification, the model was compared with a small deflection beam model, which can be found in Appendix B.

The comparison between the large deflection model and small deflection model is shown in Figure 3 through Figure 6. The comparison is for a pipe of 1” OD and 0.095” wall. Obviously the large deflection model matches small deflection model if pipe length is less than 10 ft and is either near horizontal or near vertical. And here are some remarks:

1. The two models agree with each on deflection and inclination if the pipe is not too long (less than 20 ft).
2. Small deflection model is good for bending moment prediction only if the pipe is not too long (less than 20 ft) and the pipe is initially horizontal or near horizontal.
3. For long pipe (say longer than 70 ft) the inclination at tip will be very close to 90 degrees regardless the inclination at the support. Small deflection model does not predict this.

In general, small deflection model over predicts deformation and bending moment for long beams.

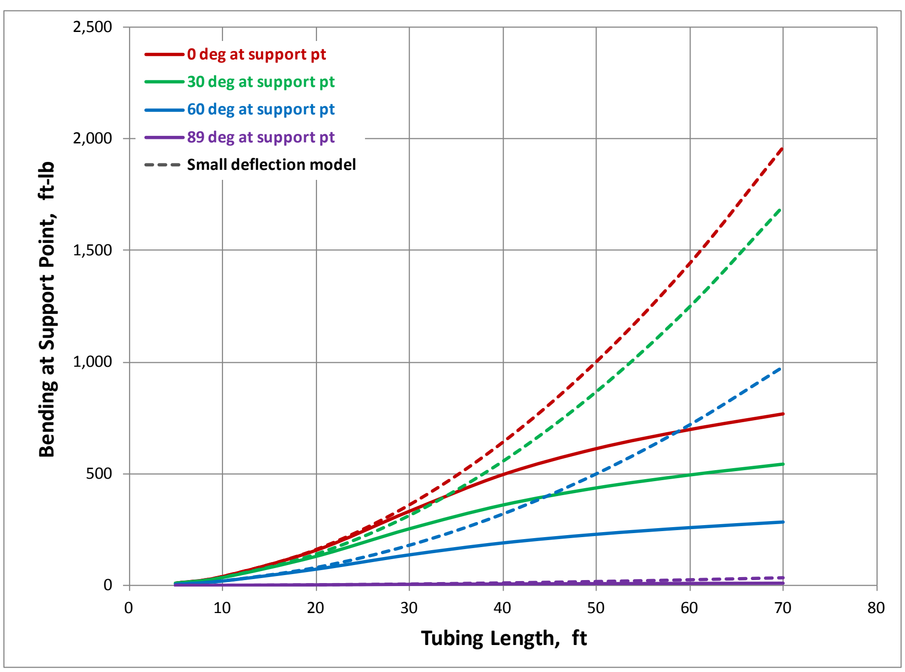


Figure 3 – Comparison of Bending Moment at Support Point between Large Deflection Model and Small Deflection Model

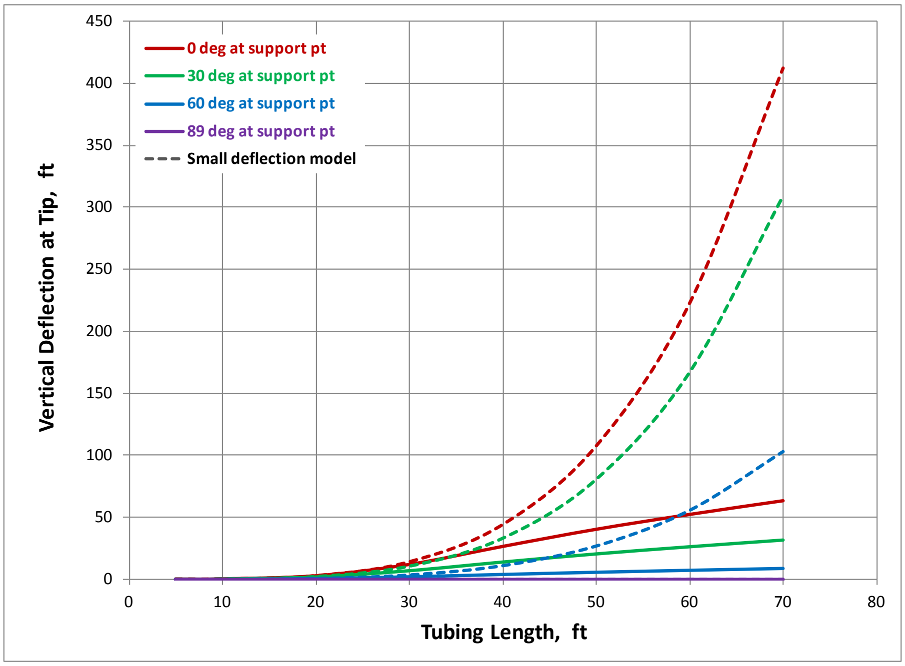


Figure 4 – Comparison of Vertical Deflection of Tip between Large Deflection Model and Small Deflection Model

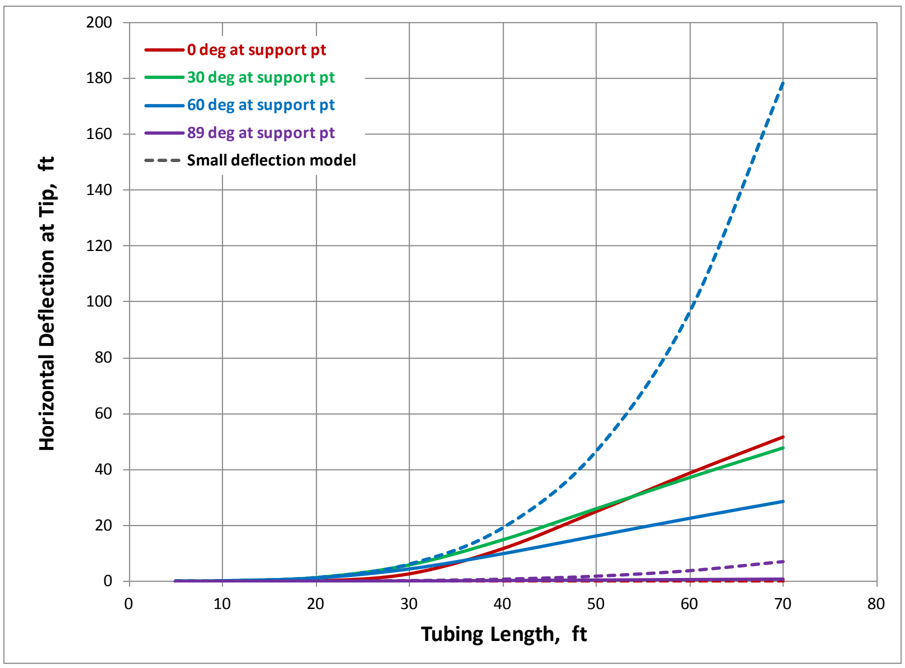


Figure 5 – Comparison of Horizontal Deflection of Tip between Large Deflection Model and Small Deflection Model

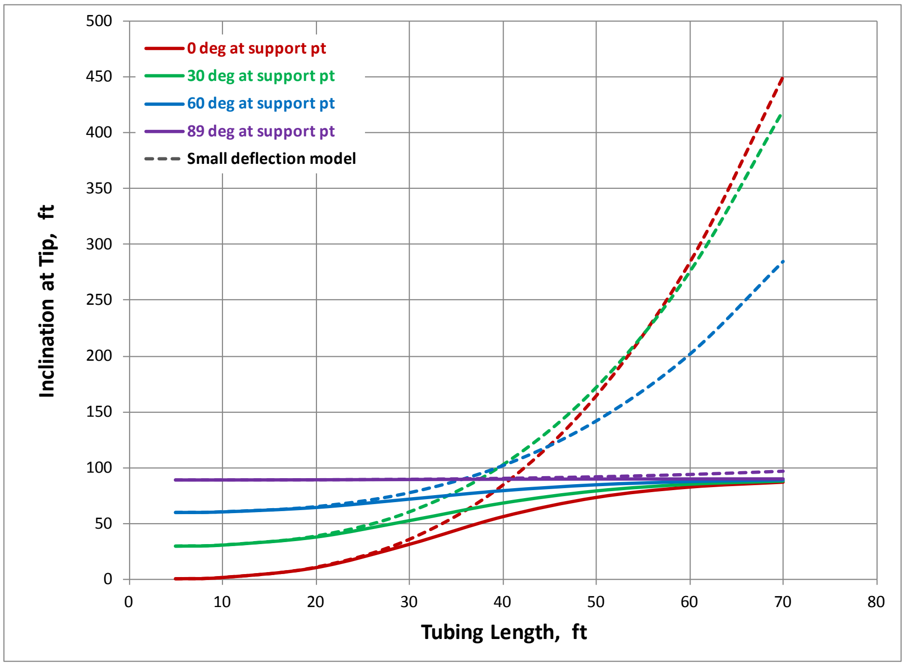


Figure 6 – Comparison of Inclination at Tip between Large Deflection Model and Small Deflection Model

Prediction of Pipe-Hole Contact in Drilling New Hole Section

The large deflection beam model was utilized in predicting pipe-hole contact during hole drilling.

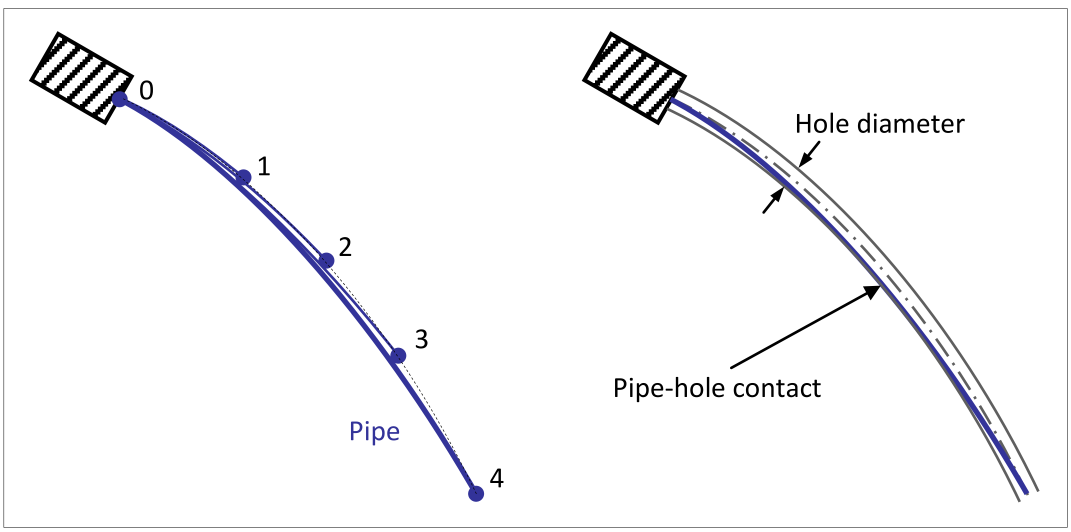


Figure 7 – Pipe-Hole Contact in Drilling New Hole Section

In Figure 7, in drilling a new hole section, the pipe is being extended. The tip of the pipe is moving from position 0 to position 4. And the trajectory of the tip forms the centerline of the new hole. Since the pipe weight is increasing as the tip moves from position 0 to position 4, the pipe will be below the centerline of the hole, except at the support point (position 0) and the tip (position 4). As the drilling go further, eventually at some point the pipe will contact the lower side of the hole at some point between the support point and the tip. And this contact point will become the next support point.

Spreadsheet Applications

Two spreadsheet applications have been designed to perform the model calculations. To use the spreadsheets, user must enable macro since the spreadsheet uses VBA code to implement the iterative procedure (see Appendix A). In the spreadsheets, the cells with yellow background color are for input.

The first spreadsheet is to make the large deflection beam calculation, as shown in Figure 8. It iterates the bending moment at support point such that the bending moment at the tip will be close to zero.

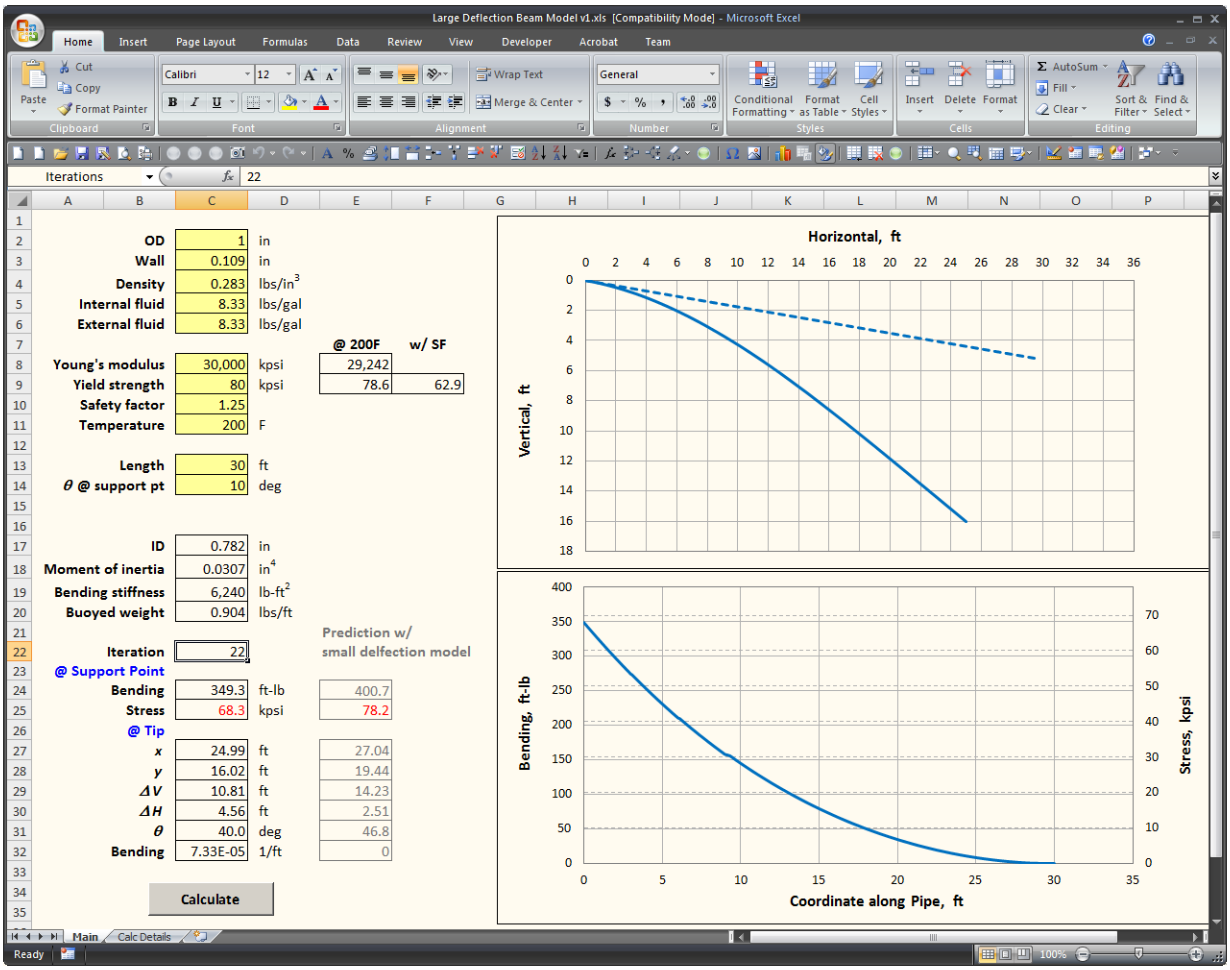


Figure 8 – Spreadsheet Application of Large Deflection Beam Model

This spreadsheet calculates

- Bending moment and stress at the support point
- Horizontal and vertical displacements of the tip relative to the support point
- Inclination at the tip
- Bending moment at the tip (This should be zero theoretically. Hence if this value is close to zero then it is a signal that a valid solution is obtained.)
- Shape of the pipe (dashed for un-bent and solid for bent) displayed in the upper chart
- Bending moment and stress along the pipe displayed in the bottom chart
- Values predicted with small deflection model just for comparison

The second spreadsheet is for predicting the pipe-hole contact in drilling, as shown in Figure 9.

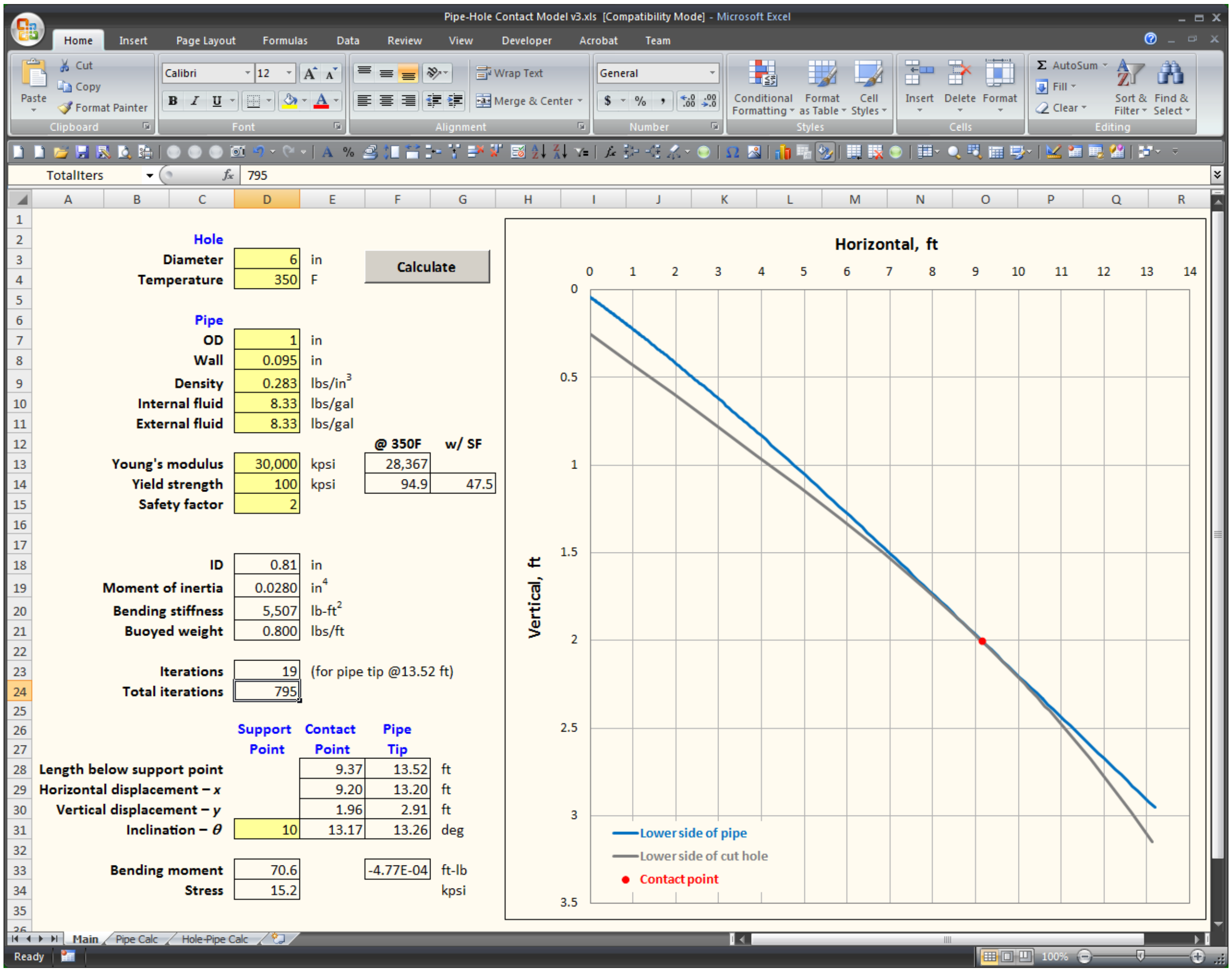


Figure 9 – Spreadsheet Application of Pipe-Hole Contact Prediction

The algorithm procedure in this application is summarized as below:

1. Extend the pipe from the support point with a constant length increment (1 ft).
2. For each position of the tip make a beam calculation. The calculated horizontal and vertical displacements of the tip will be used to define the centerline of the cut hole. Check if the pipe will exceed the hole wall along the cut hole path.
3. If the pipe exceeds the hole wall at some position in the cut hole then the current tip depth is close to the maximum tip depth and the contact point should be close to that position. Repeat step 2 for the tip depths near that position with a finer depth increment (0.1 ft) so that the final calculated contact position will have a higher accuracy.
4. Iterate the maximum tip depth such that the pipe and the cut hole will only have point contact.

Analysis Results of Inclined Cantilever Beam

The results given here were calculated with fresh water (8.33 lbs/gal) as internal fluid and external fluid.

In the figures, the arrows at the end of the curves indicate that the maximum allowed pipe length will increase rapidly as the inclination at support point approach 90 degrees (perfectly vertical).

Maximum Allowed Pipe Length – 1/2" OD

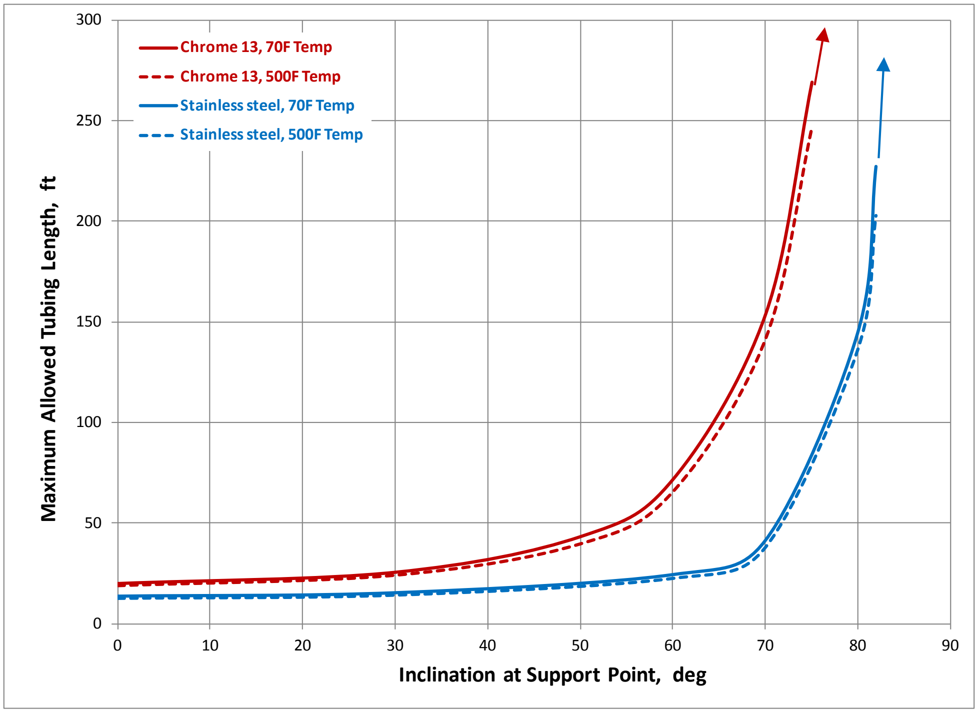


Figure 10 – Maximum Allowed Pipe Length (1/2" OD, 0.05" Wall, Chrome 13 and Stainless Steel)

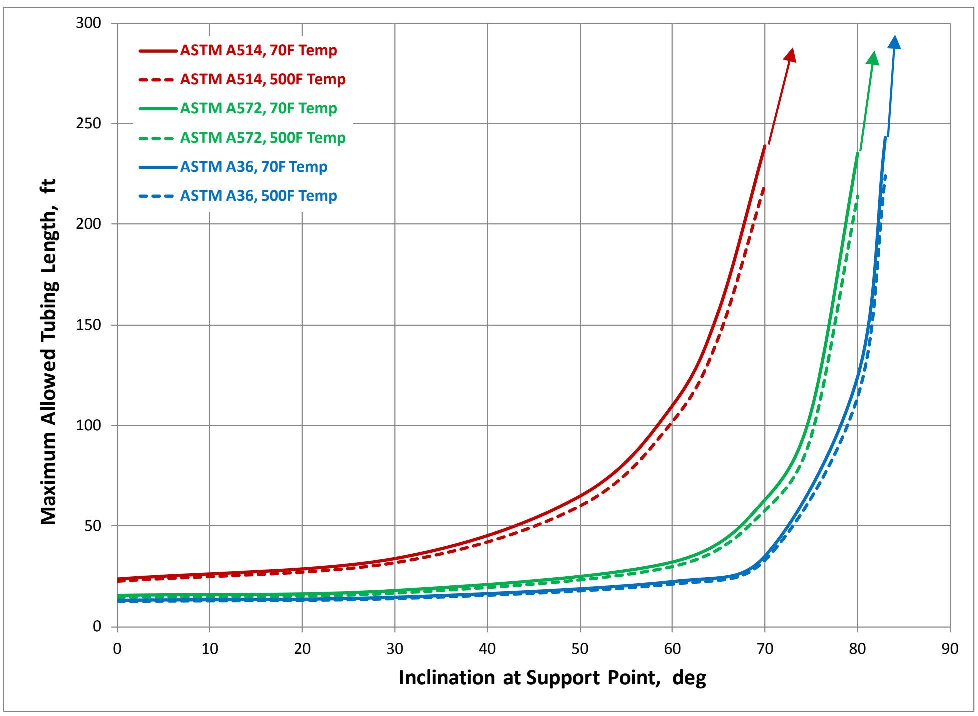


Figure 11 – Maximum Allowed Pipe Length (1/2" OD, 0.05" Wall, Standard Steels)

Maximum Allowed Pipe Length – 3/4" OD

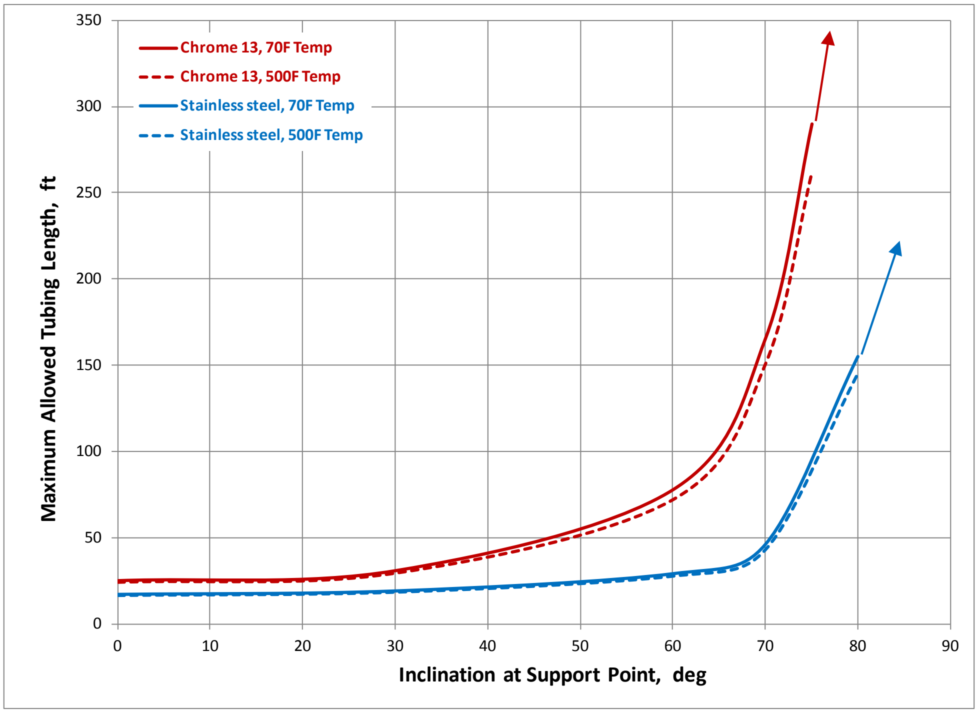


Figure 12 – Maximum Allowed Pipe Length (3/4" OD, 0.05" Wall, Chrome 13 and Stainless Steel)

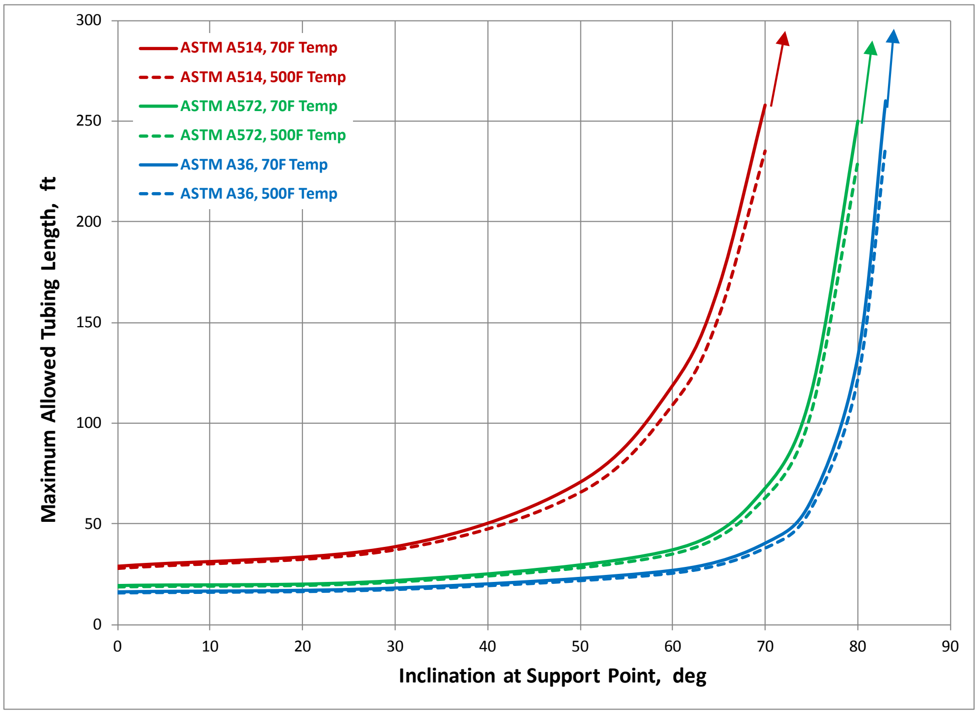


Figure 13 – Maximum Allowed Pipe Length (3/4" OD, 0.05" Wall, Standard Steels)

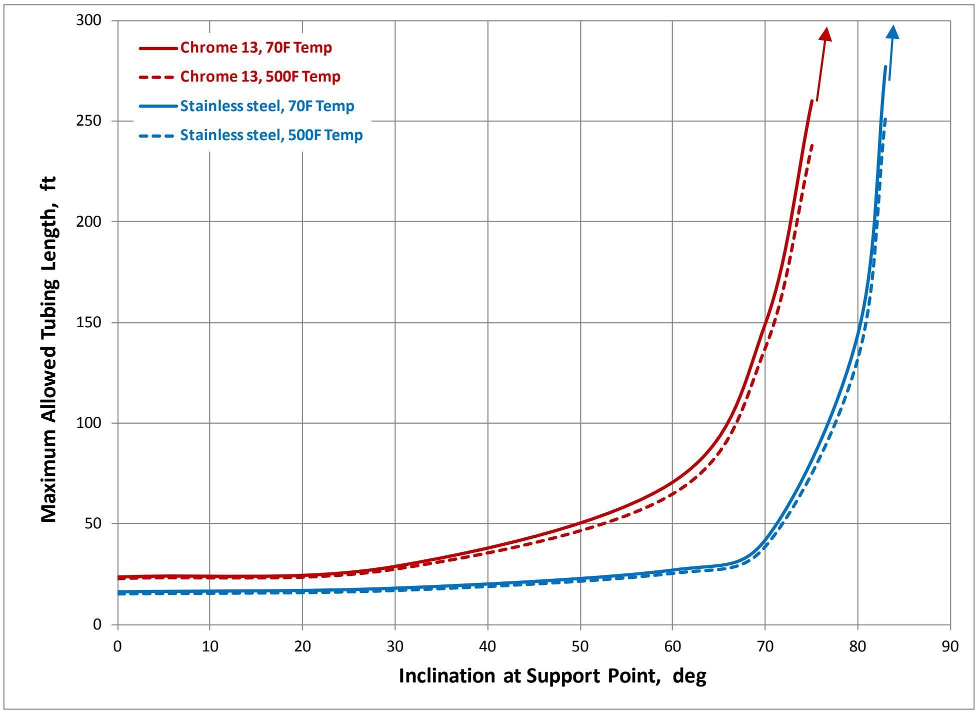


Figure 14 – Maximum Allowed Pipe Length (3/4" OD, 0.09" Wall, Chrome 13 and Stainless Steel)

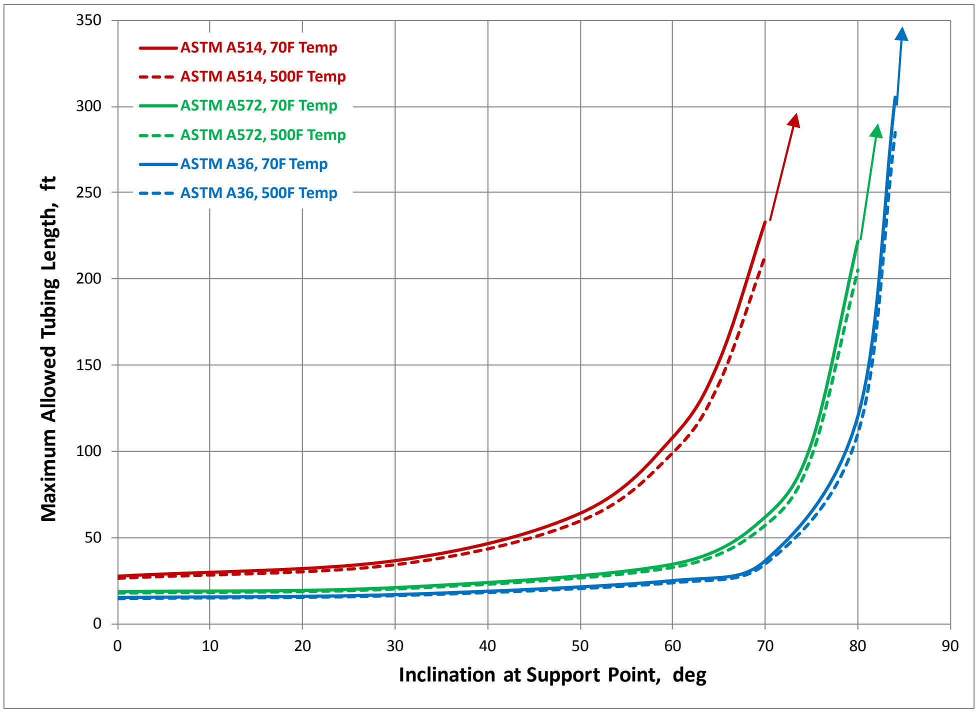


Figure 15 – Maximum Allowed Pipe Length (3/4" OD, 0.09" Wall, Standard Steels)

Maximum Allowed Pipe Length – 1" OD

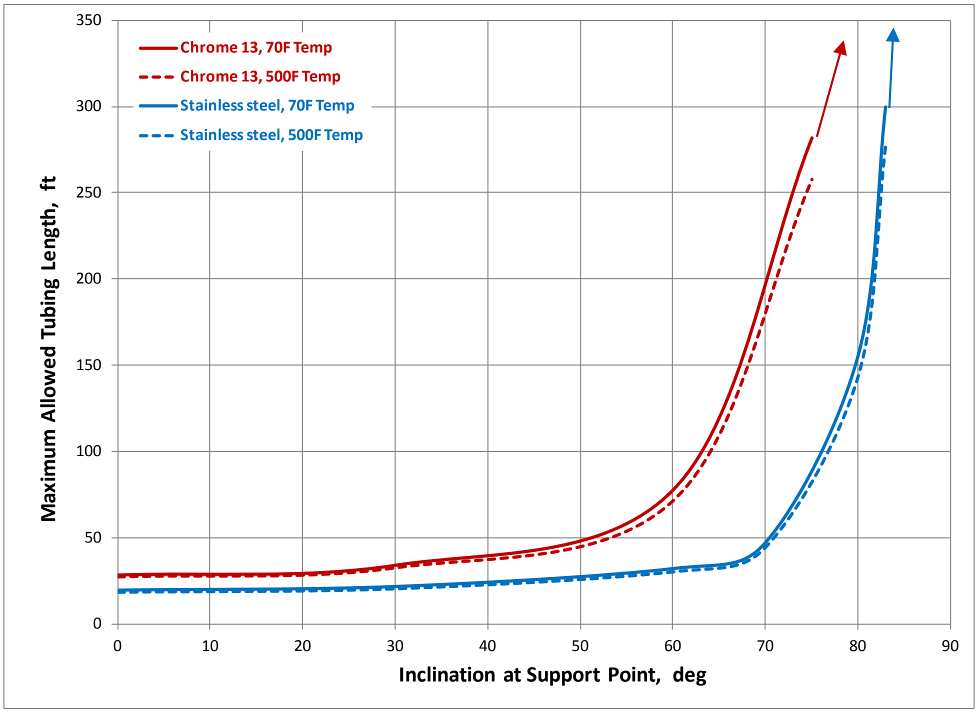


Figure 16 – Maximum Allowed Pipe Length (1" OD, 0.08" Wall, Chrome 13 and Stainless Steel)

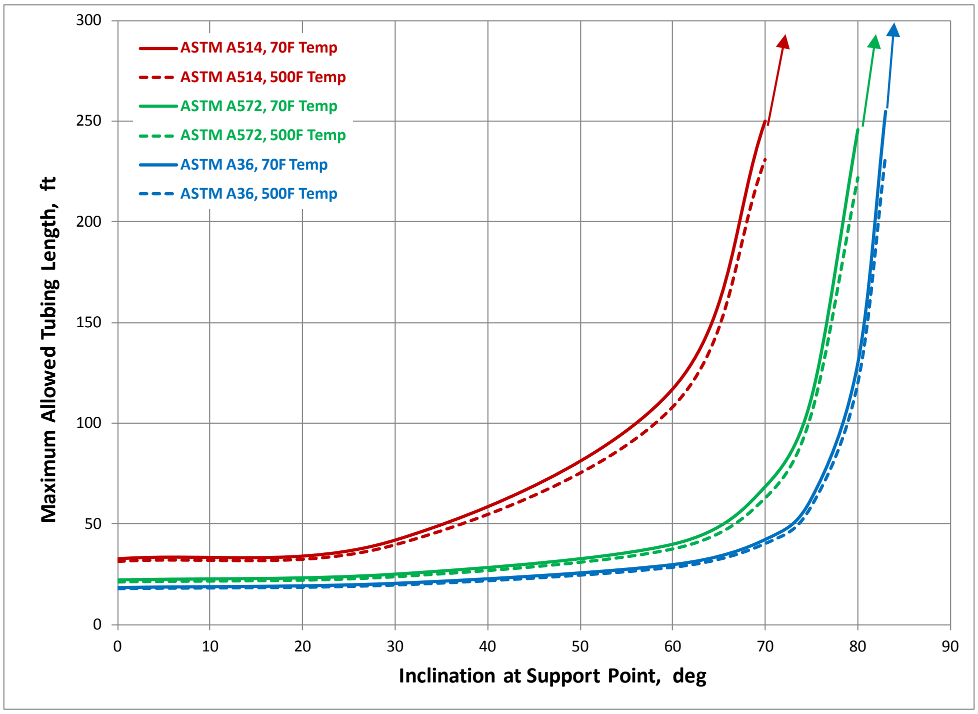


Figure 17 – Maximum Allowed Pipe Length (1" OD, 0.08" Wall, Standard Steels)

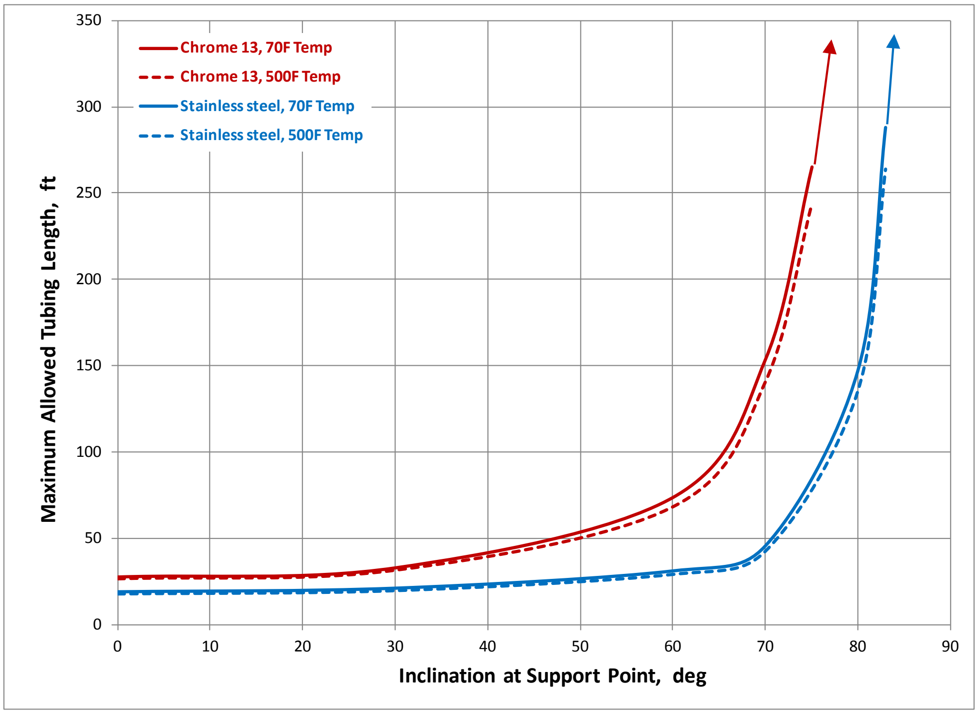


Figure 18 – Maximum Allowed Pipe Length (1" OD, 0.109" Wall, Chrome 13 and Stainless Steel)

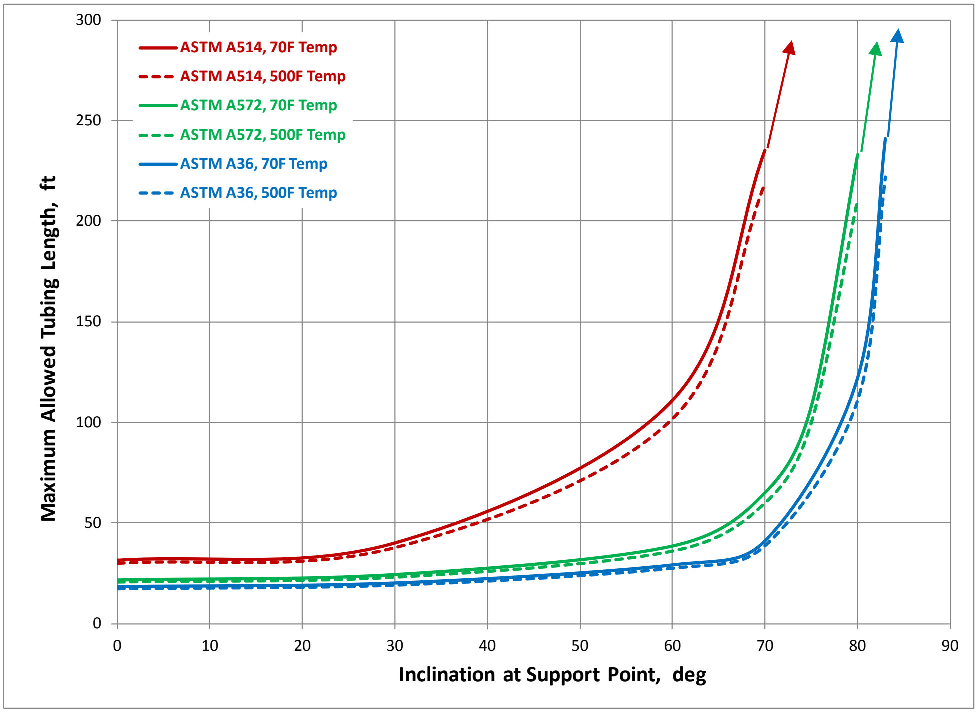


Figure 19 – Maximum Allowed Pipe Length (1" OD, 0.109" Wall, Standard Steels)

Maximum Allowed Pipe Length – 1-1/4” OD

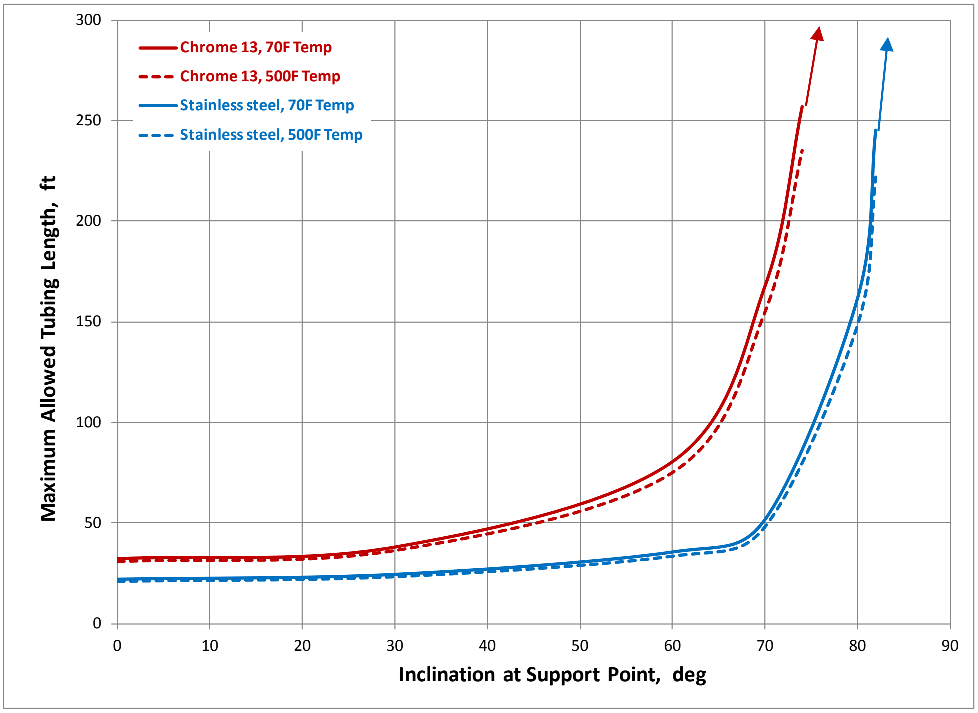


Figure 20 – Maximum Allowed Pipe Length (1-1/4” OD, 0.08” Wall, Chrome 13 and Stainless Steel)

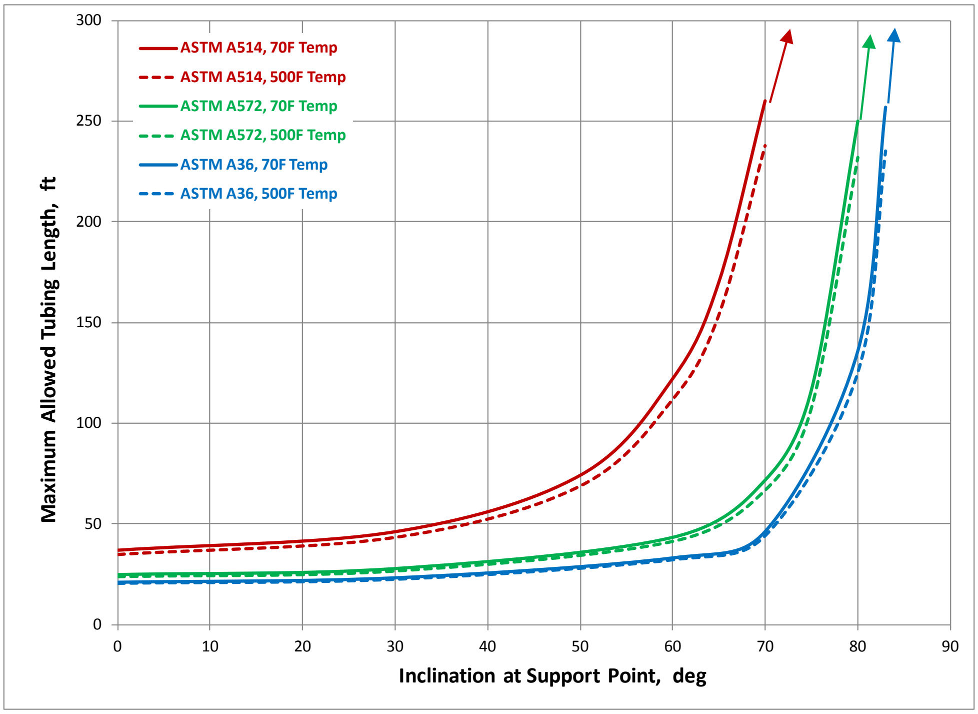


Figure 21 – Maximum Allowed Pipe Length (1-1/4” OD, 0.08” Wall, Standard Steels)

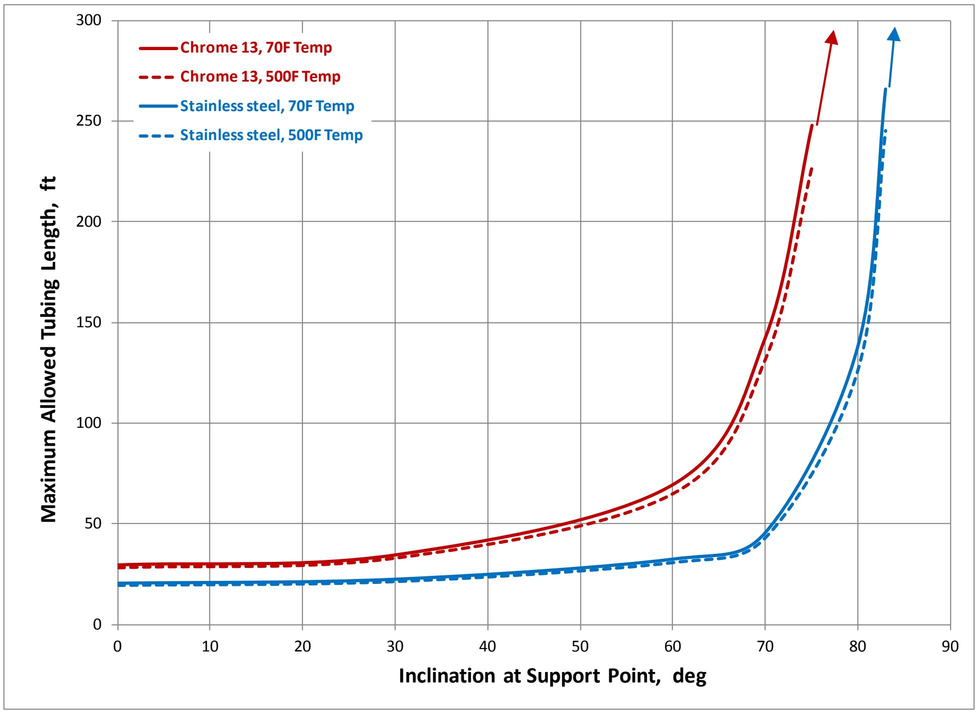


Figure 22 – Maximum Allowed Pipe Length (1-1/4” OD, 0.188” Wall, Chrome 13 and Stainless Steel)

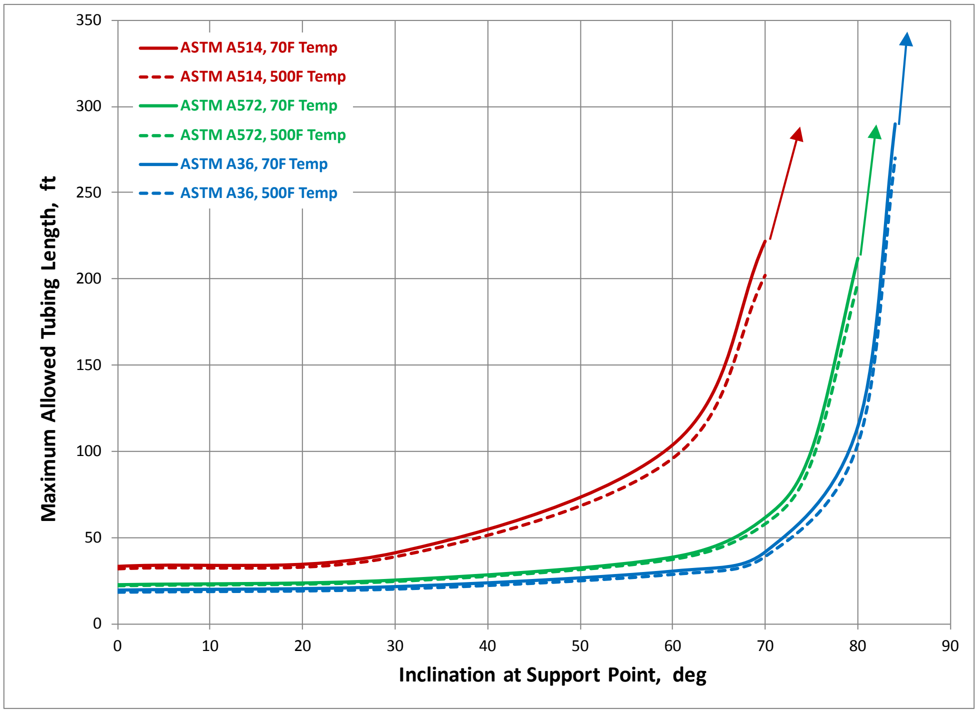


Figure 23 – Maximum Allowed Pipe Length (1-1/4” OD, 0.188” Wall, Standard Steels)

Maximum Allowed Pipe Length – 1-1/2" OD

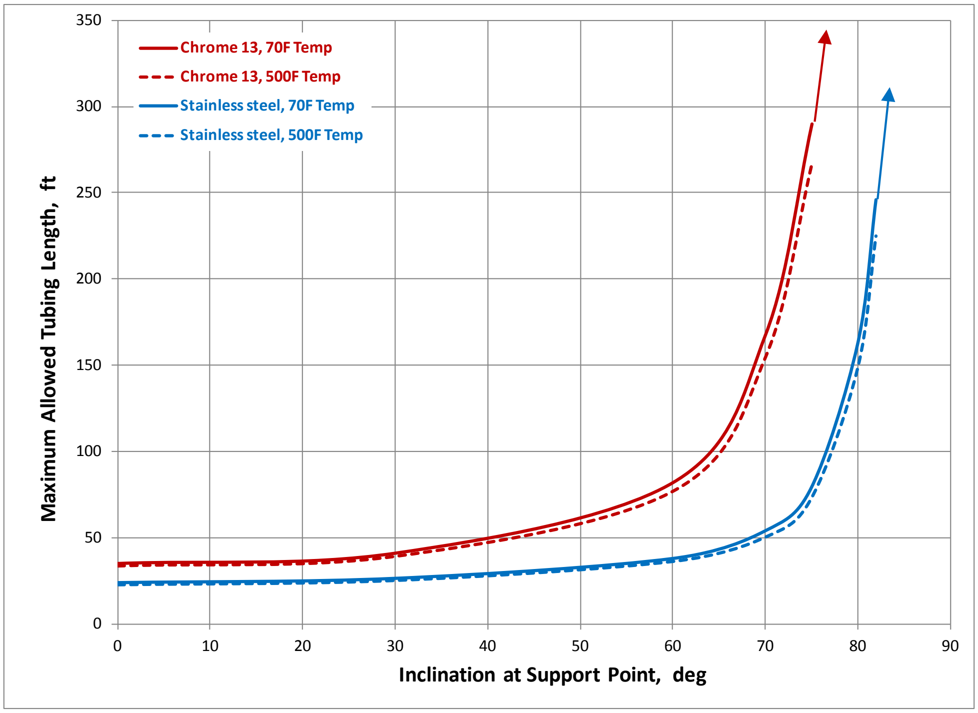


Figure 24 – Maximum Allowed Pipe Length (1-1/2" OD, 0.095" Wall, Chrome 13 and Stainless Steel)

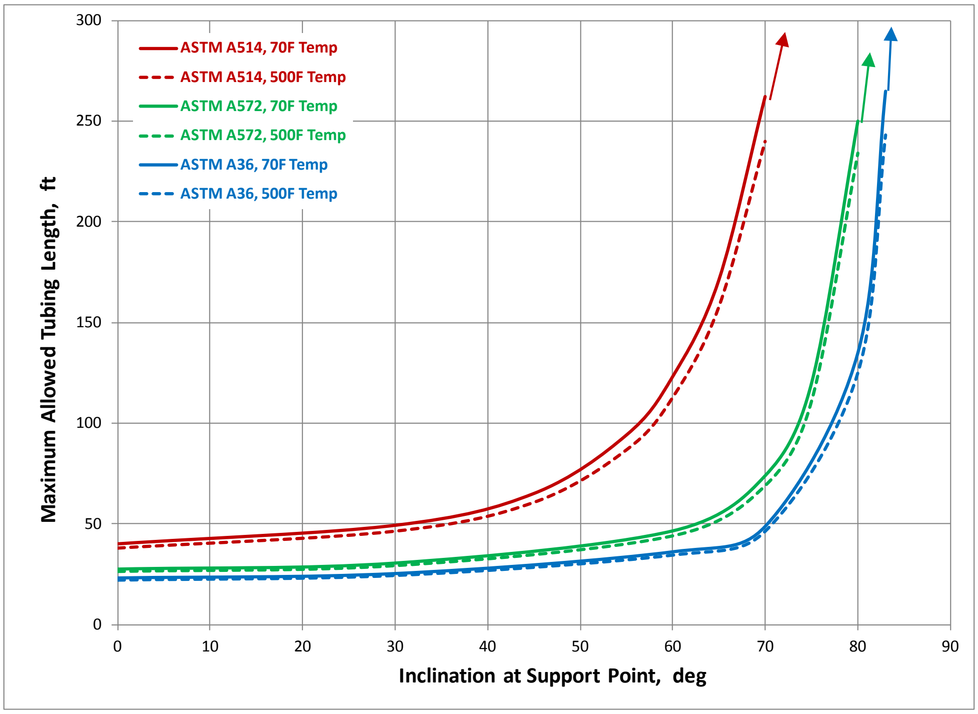


Figure 25 – Maximum Allowed Pipe Length (1-1/2" OD, 0.095" Wall, Standard Steels)

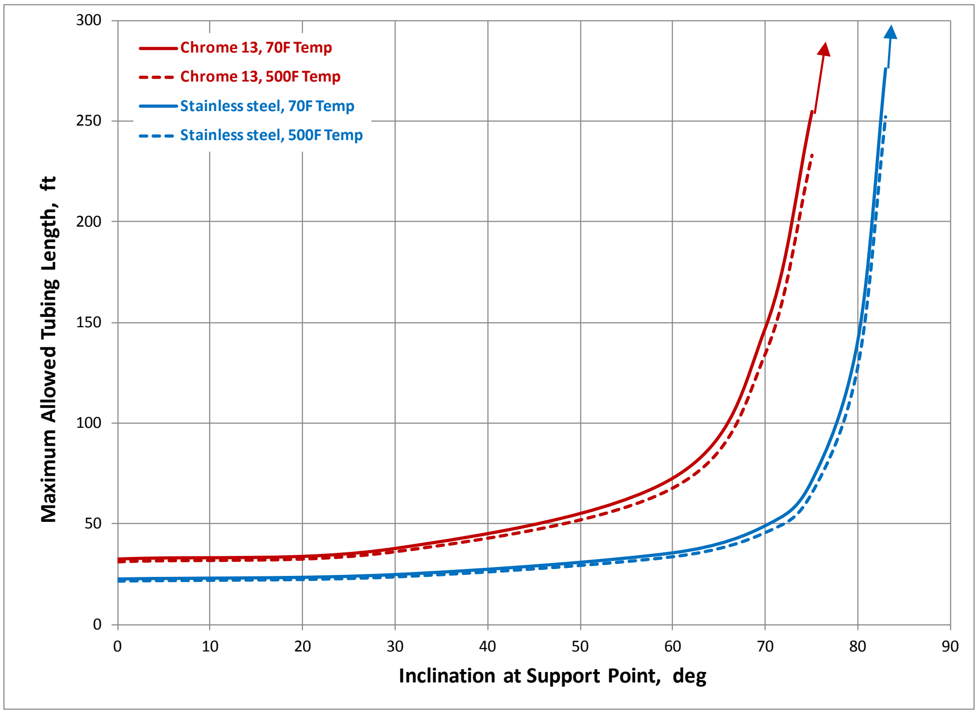


Figure 26 – Maximum Allowed Pipe Length (1-1/2” OD, 0.203” Wall, Chrome 13 and Stainless Steel)

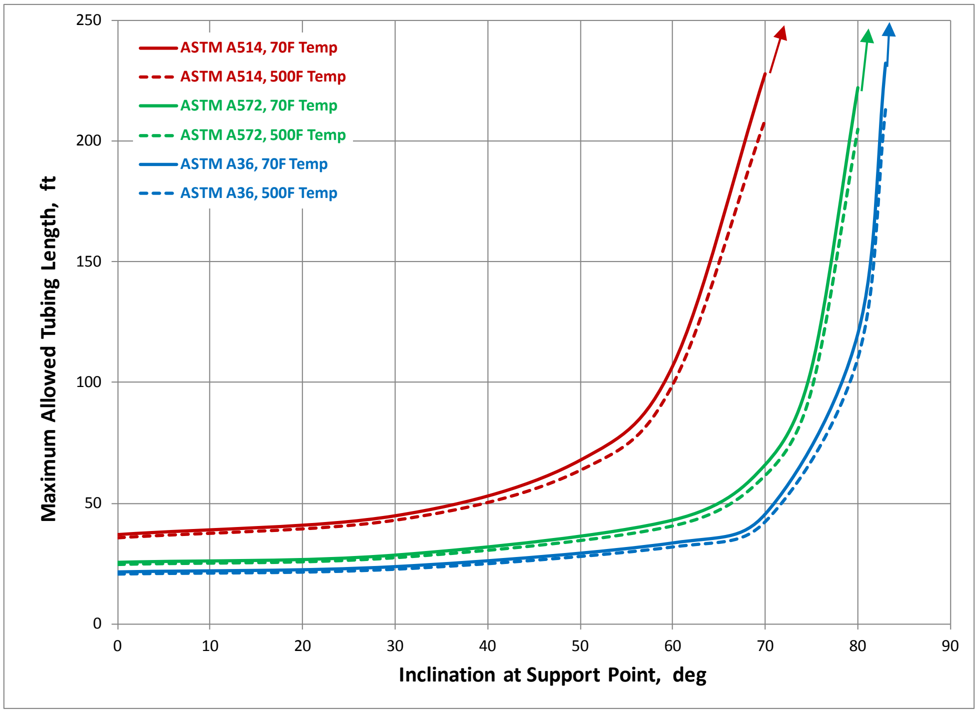


Figure 27 – Maximum Allowed Pipe Length (1-1/2” OD, 0.203” Wall, Standard Steels)

Displacements and Inclination at Tip

Charts of displacements and inclination at tip can be made with the spreadsheet calculation. An example is shown below for the Pipe of 1” OD, 0.08” wall, and stainless steel as material.

In the figures, the X symbols at the end of the curves indicate the maximum allowed pipe length.

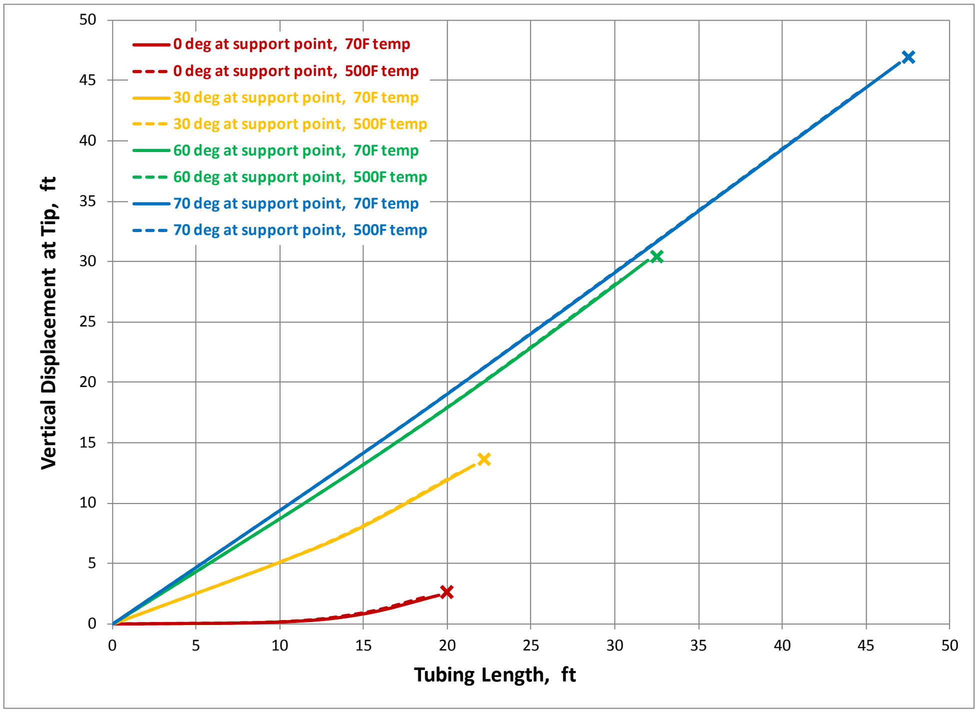


Figure 28 – Vertical Displacement at Tip (1” OD, 0.08” Wall, Stainless Steels)

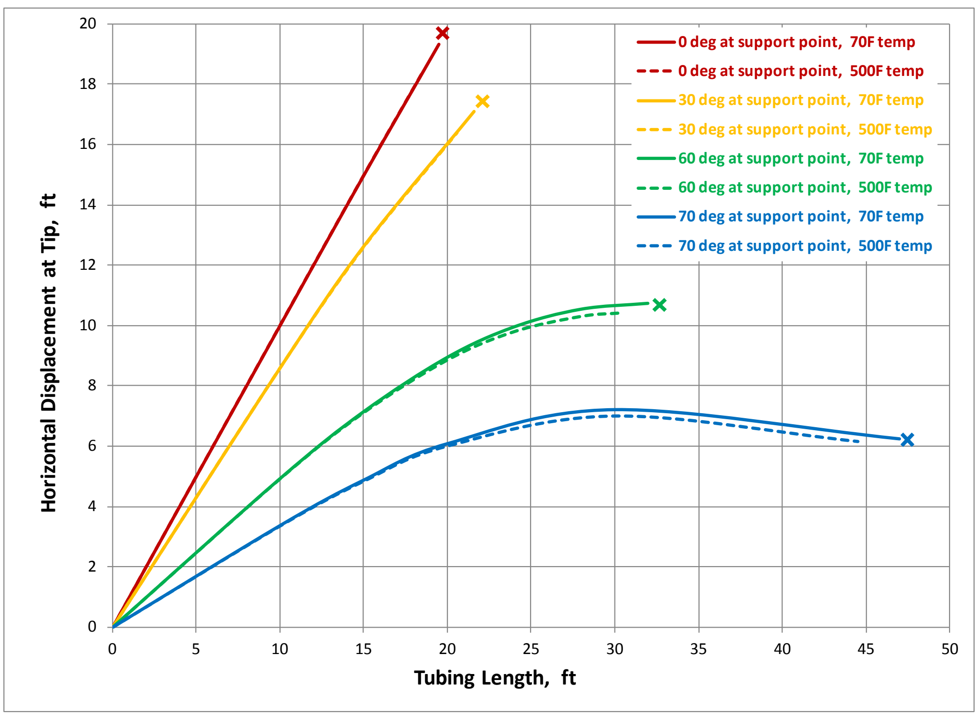


Figure 29 – Horizontal Displacement at Tip (1” OD, 0.08” Wall, Stainless Steels)

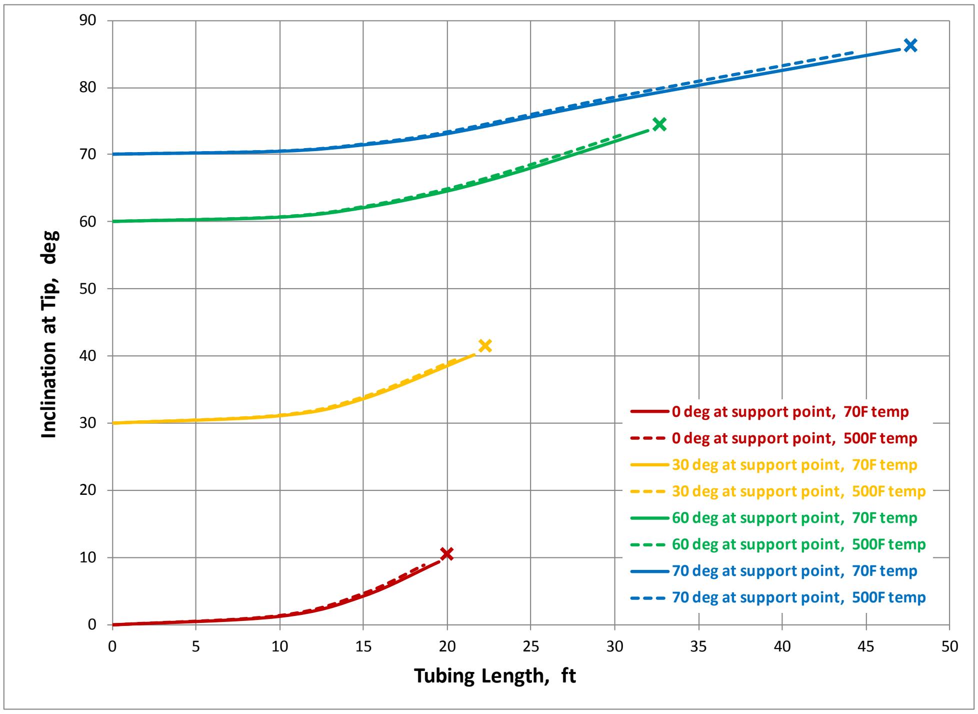


Figure 30 – Inclination at Tip (1” OD, 0.08” Wall, Stainless Steels)

Conclusions

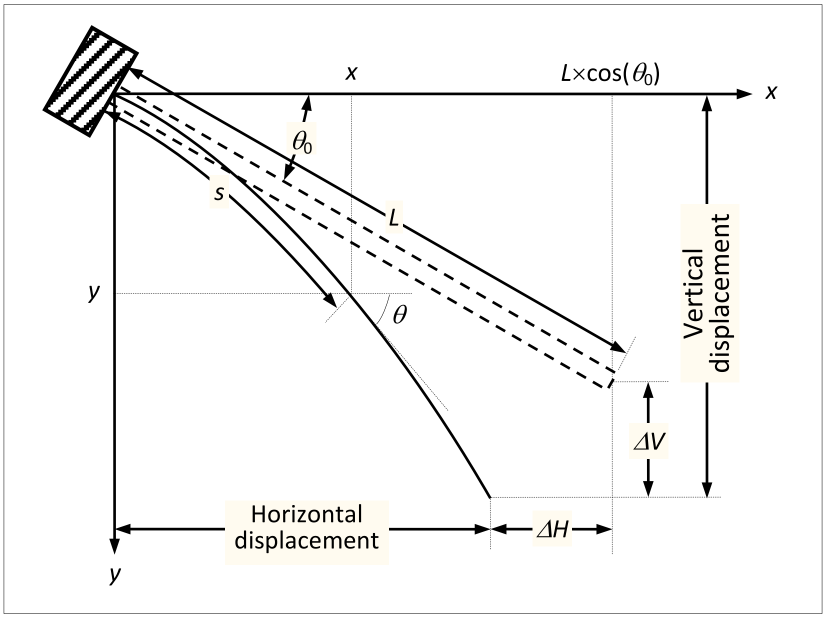
The following conclusions are made as a result of this analysis:

1. The large deflection beam model and the first spreadsheet application developed in this study can be used to calculate (1) vertical displacement, horizontal displacement and inclination of the Pipe tip; (2) bending moment and stress along the pipe; and (3) the shape of the pipe.
2. The model can be used to give maximum allowed pipe length without yielding the pipe.
3. The spreadsheet application has a limitation in calculating the maximum allowed pipe length for nearly vertical pipe since the maximum length will approach infinite as the inclination at the support point approach 90 degrees. In this study it was found the maximum inclination at the support point is about 85 degrees.
4. The second spreadsheet application can be used to predict the pipe-hole contact during a new hole section drilling. This can be used to determine when the support point need be moved forward.

Appendix A – Large Deflection Cantilever Beam Model

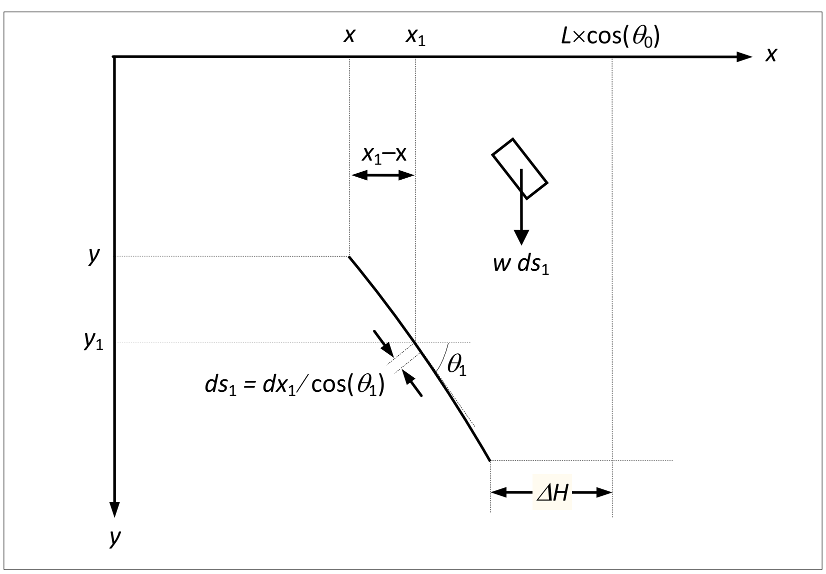
Assume an inclined cantilever beam is fixed at the left end, as shown below, with the following properties:

- L – Beam length, ft
- θ_0 – Inclination at the support point, deg
- w – Buoyed weight, lbs/ft



The curvature of the beam can be stated in terms of arc length, s, and slope angle, θ. The product of bending stiffness, EI, and the curvature of the beam equals the bending moment:

$$EI\frac{d\theta}{ds} = M$$



Consider the bending moment at *x* cause by the pipe length right to *x*:

$$M = \int_{x}^{L\cos\theta_0 - \Delta H} \frac{w(x_1 - x)}{\cos\theta_1} dx_1$$

Therefore

$$M = w\int_{x}^{L\cos\theta_0 - \Delta H} \frac{x_1}{\cos\theta_1} dx_1 - wx(L - s)$$

$$\frac{dM}{ds} = -w(L - s)\cos\theta$$

Generally it is difficult to obtain the analytical solution of the above equation. In this study fourth-order Runge-Kutta is adopted to obtain the solution numerically.

The numerical solution is outlined as follows:

$$\begin{cases} \dfrac{d\theta}{ds} = \dfrac{M}{EI} \\ \dfrac{dM}{ds} = -w(L - s)\cos\theta \end{cases}$$

with the boundary conditions

$$\begin{cases} \theta = \theta_0 & \text{at } s = 0 \\ M = 0 & \text{at } s = L \end{cases}$$

And *x*, *y* coordinates at s can be calculated as follows:

$$x = \int_0^s \cos\theta\, ds$$

$$y = \int_0^s \sin\theta\, ds$$

Note that Runge-Kutta solution of the above system depends on the values of θ_0 and M_0 at $s = 0$. However, M_0 is unknown. Hence iteration must be used to obtain M_0 using the condition $M = 0$ at $s = L$.

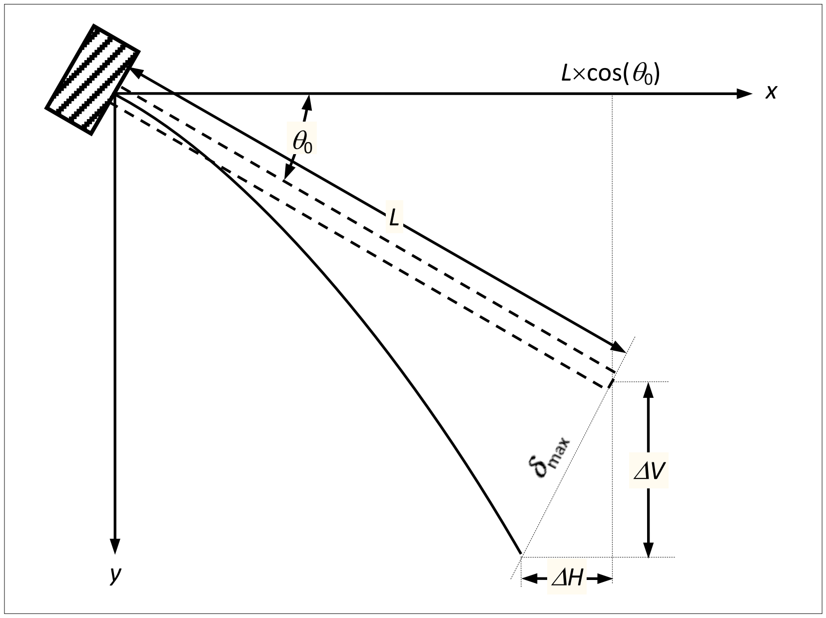
Appendix B – Small Deflection Cantilever Beam Model

With small deflection model of a horizontal cantilever beam with self weight,

$$\delta_{\max} = \frac{wL^4}{8EI}$$

$$\theta_{\max} = \frac{wL^3}{6EI}$$

$$M_0 = \frac{1}{2} w L^2$$



When applied to an inclined cantilever beam, the effective weight will be $w \times \cos\theta$. And the deflection, inclination, and bending moment can be calculated as follows

$$\Delta V = \frac{w(\cos\theta_0)^2 L^4}{8EI}$$

$$\Delta H = \frac{w \sin\theta_0 \cos\theta_0 \, L^4}{8EI}$$

$$\theta_{\max} = \theta_0 + \frac{wL^3}{6EI}$$

$$M_0 = \frac{1}{2} w L^2 \cos\theta_0$$

Note that, with small deflection model, the horizontal deflection of the tip is solely caused by the rotation of $\delta_{\max}$. If the pipe is initially horizontal (θ_0 = 0) then there is no horizontal deflection.