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PRELIMINARY THERMAL ANALYSIS OF BINARY CYCLE FOR GEOTHERMAL PILOT PLANT

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ABSTRACT

On the basis of the criteria set forth, the refrigerants as a group are found to have desirable characteristics as working fluids for the binary system. Three fluids, Freon-12, Freon-113, and isobutane are chosen for more detailed analysis. (Fluid properties deviate considerably from the perfect gas laws; therefore C_p, the specific heat at constant pressure, varies considerably and cannot be assumed constant.) It appears that these fluids have geothermal brine utilization factors, or effectiveness in the range of 20 Btu produced per pound of brine circulated. The thermal efficiency of the cycle is found to be 10-20%. Flow rates range from 10^6 to 10^7 lb/hr for the working fluid, for the condensing water, and for the brine.

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I. INTRODUCTION

The harnessing of geothermal energy for electrical power is a relatively recent undertaking. Engineering research and development are necessary before geothermal energy can be profitably integrated into regional power networks. One such avenue of development is the design and construction of a demonstration geothermal power plant in the Raft River Valley[1].

The Raft River project is concerned with comparing the relative merits of two thermodynamic cycles, a steam cycle and a binary cycle. Preliminary studies have been completed on both cycles, and design work has begun on a steam cycle. The intent of this analysis is to further study thermodynamic features of a binary cycle and to determine parameters for choosing a working fluid.

II. DEFINITION OF THE BINARY CYCLE

A binary vapor cycle is one that employs two different working fluids. Working fluids are those which are used for the production of power. By this definition, the binary cycle under consideration is not a true binary cycle. The intent is to utilize the geothermal hot water as a heat source for the actual working fluid. (There is also an option in the proposed project to combine the steam cycle and the "binary" cycle to form a true binary cycle, in which both fluids are used to provide power.) Therefore, the binary cycle in question uses two fluids, but only one of them is working fluid.

III. WORKING FLUID SELECTION CRITERIA

There are three major areas of concern in the selection of a working fluid. These are:

(1) The thermodynamic properties of the fluid as they relate to design considerations

(2) The physical properties of the fluid as they relate to expense, safety, and durability

(3) The thermodynamic behavior of the fluid as it is utilized to produce power.

1. THERMODYNAMIC WORKING FLUID PROPERTIES

The effectiveness of power-producing systems greatly depends on the suitability of the working fluid to the hardware involved. These are the turbine, pumps, piping, and heat exchangers, including boiler, condenser, heater, superheater, and regenerator. Design considerations of these power-producing mechanisms dictate certain desirable working fluid properties. These are:

(1) High liquid density to minimize pipe size and pumping requirements.

(2) Low latent heat of vaporization to minimize boiler and condenser size.

(3) High gas specific volume to maximize turbine output.

(4) Low gas specific volume to minimize piping and turbine sizes. (Items 3 and 4 are contradictory and optimization with respect to specific volume is required.)

(5) High molecular weight (which is inversely proportional to gas specific volume) to minimize blade speed for the turbine.

(6) High specific heat at constant pressure to minimize the size of the heat exchangers and to minimize the flow rate required through them.

(7) High film coefficient for more efficient heat exchangers. Thermal conductivity and density are two indicators of this parameter. The brine side of the HX or the metal resistance may be the controlling factor and the heat exchanger capacity may be insensitive to this working fluid property.

(8) Pressure of vaporization (corresponding to a temperature near that of the source) should be near the top of the dome to maximize superheat capability and cycle efficiency. Toward the top of the dome, heat of vaporization is relatively small (boiler size minimized), and the turbine pressure ratio is maximized.

(9) Pressure of condensation should be near atmospheric to reduce stress in all components. At very low pressures, the specific volume is large, requiring larger piping and components. Ejectors and additional equipment are required for pressure lower than atmospheric.

(10) The condition of the fluid at the exit of the turbine should be very close to a saturated vapor. Two-phase flow requires a less efficient condensing, and superheated vapor requires that regeneration be used to recover excess superheat.

(11) Low vapor pressure for lower pressure system.

(12) Low melting point (preferably less than -30°F @ atmospheric pressure) so that if the plant is shut down during winter, the working fluid will not solidify.

(13) High decomposition temperature (greater than 400°F @ atmospheric pressure) so that the fluid will not dissociate at cycle temperatures.

As a group, the fluids used as refrigerants fulfill these requirements. Nine fluids have been selected for comparison. See Tables I and II for tabulation of the properties listed above.

TABLE I

SIGNIFICANT THERMODYNAMIC PROPERTIES

	Temperature = 80°F				Temperature = 290°F		
Working Fluid	P_{SAT} (psia)	Δh_{VAP} (Btu/lb)	ρ_{LIQ} (lb/ft^3)	ρ_{GAS} (lb/ft^3)	P_{SAT} (psia)	Δh_{VAP} (ft^3/lb)	v_{GAS} @ SAT (ft^3/lb)
Butane	37.6	155.3	33.78	0.85	400	80	0.16
Carrene-7	117.2	71.8	71.95	2.37	cr	cr	cr
Freon-11	16.3	77.8	91.88	0.40	270	51	0.16
Freon-12	98.9	58.9	81.45	2.43	cr	cr	cr
Freon-21	28.0	99.9	85.03	0.52	450	53	0.09
Freon-22	159.7	77.9	74.12	2.89	cr	cr	cr
Freon-113	6.9	65.9	97.45	0.23	155	43	0.19
Freon-114	33.0	55.1	90.56	1.05	460	15	0.04
Isobutane	53.9	140.5	34.35	0.89	cr	cr	cr

TABLE II

SIGNIFICANT THERMODYNAMIC PROPERTIES

Working Fluid	Mol Wt	Critical Temp (°F)	Critical Pressure (psia)	Melting Point @ ATM (°F)	Cp @ 80°F, P_{SAT} (Btu/lb-°F)	k @ 80°F (GAS) (Btu/ft-hr-°F)	Decomp Temp (°F)	$\frac{P_{SAT} @ 290°F}{P_{cr}}$
Butane	58.12	306.0	550.1	-211	0.40	0.0092	1000	0.727
Carrene-7	99.29	221.1	631.0	-254	0.18	0.0053	$\geq$400	cr
Freon-11	137.4	388.4	635.0	-168	0.15	0.0043	$\geq$400	0.425
Freon-12	120.9	232.7	582.0	-252	0.18	0.0055	$\geq$400	cr
Freon-21	102.9	353.3	750.0	-211	0.15	0.0057	$\geq$400	0.600
Freon-22	86.50	204.8	716.0	-256	0.18	0.0068	$\geq$400	cr
Freon-113	187.4	417.4	495.4	-31	0.18	0.0045	$\geq$400	0.313
Freon-114	170.9	294.3	474.0	-137	0.18	0.0052	$\geq$400	0.970
Isobutane	58.12	272.7	537.0	-229	0.40	0.0094	1000	cr

2. PHYSICAL PROPERTIES

For economic and safety reasons, four criteria will apply to the fluids under consideration. The fluid should be:

(1) Nonflammable or of low flammability to reduce the danger of fire or explosions

(2) Noncorrosive to be compatible with standard engineering materials

(3) Nontoxic to avoid danger of injury to personnel in case of a leak

(4) Miscible with oil and lubricants to keep the system lubricated and clean.

See Table III for a tabulation of these criteria as they apply to the fluids in question.

A further consideration is the cost and availability of the fluids. The price must be evaluated on the basis of the cost to fill the system.

TABLE III

PHYSICAL PROPERTIES

Working Fluid	Flammability	Corrosiveness	Lubricant Miscibility	Toxicity
Butane	High	Noncorrosive	Good	Slight
Carrene-7	Nonflammable	Noncorrosive	Good	Slight
Freon-11	Nonflammable	Noncorrosive	Good	Low
Freon-12	Nonflammable	Noncorrosive	Good	Very Slight
Freon-21	Nonflammable	Noncorrosive	Good	Low
Freon-22	Nonflammable	Noncorrosive	Fair	Slight
Freon-113	Nonflammable	Noncorrosive	Good	Low
Freon-114	Nonflammable	Noncorrosive	Fair	Very Slight
Isobutane	High	Noncorrosive	Good	Very High

3. PRELIMINARY THERMODYNAMIC ANALYSIS

For a first look at working fluids, an "ideal cycle" analysis is conducted. The most efficient cycle for power production was found to be Rankine Cycle. The requirements for the Raft River Pilot Plant state that 10 MW(e) will be produced. For a generator efficiency of 90%, this fixes turbine output at 11 MW (th) or 3.8 x 10^7 Btu/hr. All other efficiencies are assumed to be 100%. The geothermal hot water (brine) is expected to arrive at 300°F and 69 psia. Assuming a 10° final temperature difference for the heat exchanger, the working fluid can be heated to 290°F. It is then expanded isentropically through a turbine with a pressure ratio of ~10. The amount of superheat is minimized to minimize regeneration. Condensing temperature is 80°F, and condensing pressure is at or above atmospheric when possible. Maximum cycle pressure is kept to a minimum.

Two good indicators of cycle behavior are the thermal efficiency and the net work available. The net work is the turbine work minus the pump work. Thermal efficiency η_{et}, is defined as the ratio of the net work to the amount of energy input required to produce that work.

For those cycles which have heat available for regeneration, thermal efficiency is calculated for two cases. First, the case of neglecting the available heat from regeneration is computed as above, then the efficiency of a cycle utilizing that energy is found. For regeneration, η_{et} is defined as the ratio of the net work to the heat input minus the heat supplied by regeneration. Less energy is required since some heat can be provided internally.

Heat input and output is calculated using the published fluid enthalpy at each state point. See Table IV for a summary of the results.

TABLE IV

IDEAL CYCLES

Working Fluid	η_{TH} (no Regen) (%)	Net Work $\left(\frac{Btu}{lb}\right)$	$\frac{Btu}{lb}$'s Avail for Regen	η_{TH} (with Regen) (%)	Turbine Pressure Ratio	Max Cycle Pressure (psia)	Condenser Pressure (psia)
Butane	14.4	33.3	31	16.4	10	370	37
Carrene-7	15.3	13.0	0	--	7.7	900	117
Freon-11	19.9	20.7	5	21	10	160	16
Freon-11	21.9	21.6	0	--	15.5	250	16
Freon-12	19.6	14.4	0	--	8	800	100
Freon-21	19.7	24.5	0	--	9	250	28
Freon-22	17.1	18.1	9	18.8	4.4	700	160
Freon-22	19.5	18.9	0	--	6.3	1000	160
Freon-113	17.7	15.8	10	20.2	10	140	14
Freon-113	21.5	20.8	9	24.1	20	140	7
Freon-114	15.5	15.4	14	18.7	10	330	33
Isobutane	19.5	40.7	27	22.4	8.9	470	53
Isobutane	18.7	35.3	12	20	10.4	550	53

IV. PRELIMINARY WORKING FLUID SELECTION

On the basis of the three groups of criteria outlined in Sections III-1, -2, and -3, three fluids are chosen for further study. These are isobutane, Freon-12, and Freon-113. The other fluids have properties such as low efficiency, low net work output, very high latent heat of vaporization, very high vapor pressure, etc., which disqualify them. The fluids chosen appear to be good compromises with respect to listed criteria. Isobutane has a very high net work output, Freon-12 appears to be applicable for a supercritical cycle (low critical temperature), and Freon-113 has a high molecular weight and relatively low latent heat of vaporization.

A more detailed analysis of these three fluids follows.

1. VARIATIONS OF THE CYCLE

As mentioned in Section III-3, the thermodynamic cycle which has been chosen for geothermal power production is the Rankine Cycle. There are three variations on this cycle: (a) No superheat, or turbine inlet temperature equals boiler outlet temperature, (b) Some amount of superheat, or turbine inlet temperature is greater than boiler outlet temperature, and (c) Supercritical, or no boiler required. All three of these options can be used with or without regeneration. See Figure 1 and Figure 2. The listings (a), (b), and (c) correspond to the discussion above.

2. EQUATIONS OF STATE

For a more complete study, flow rates of the working fluid, condensing water, and brine must be calculated. Efficiencies for components of the system must be accounted for.

In order to do this analytically, some assumptions are made. First, the fluid vapor must behave like a perfect gas and second, C_P, the specific heat at constant pressure, is constant. If these are good assumptions, equations can be derived from the perfect gas law to describe fluid parameters for the cycle. These equations, with a definition of terms, follow. The derivation of the equations is found in the Appendix.

2.1 Definition of Terms

C_{PG} — The specific heat at constant pressure for the working fluid in the vapor region Btu/lb-^{o}R

C_{PL} — The specific heat at constant pressure for the working fluid in the liquid region Btu/lb-^{o}R

E — The effectiveness of the cycle (Btu/lb of brine)

Δh_B — The latent heat of vaporization of the working fluid across the boiler (Btu/lb)

Δh_{COND} — The latent heat of vaporization of the working fluid across the condenser (Btu/lb)

Δh_R — The heat available for regeneration (Btu/lb)

Δh_T	The total external heat input to the cycle (Btu/lb)
k	The ratio of specific heats for the working fluid
P_{IN}	The turbine inlet pressure (psia)
P_{OUT}	The turbine outlet pressure (psia)
ΔP_{PI}	The absolute value of the pressure drop across the condenser (psid)
ΔP_P	The absolute value of the pressure drop across the boiler (psid)
T_B	The boiler temperature (^oR)
T_{BI}	The initial or inlet brine temperature (^oR)
T_{BO}	The final or outlet brine temperature (^oR)
T_{COND}	The condenser temperature (^oF)
T_{IN}	The turbine inlet temperature
ΔT_{COND}	The temperature drop of the brine across the condenser
ΔT_P	The pinch point (^oF)
ΔT_S	The amount of superheat (^oF)
ΔT_2	The brine outlet temperature minus the working fluid temperature at the entrance to the heater (^oF)
v_L	The specific volume of the working fluid in the liquid region $\left(\frac{ft^3}{lb}\right)$
W_N	The net power out of the turbine (Btu/hr)
$\dot{w}$	The working fluid flow rate $\left(\frac{lb}{hr}\right)$
$\dot{w}_B$	The brine flow rate (lb/hr)
$\dot{w}_C$	The condensing water flow rate $\left(\frac{lb}{hr}\right)$
η_{et}	The thermal efficiency of the cycle (%)
η_P	The pump efficiency (%)
η_T	The turbine efficiency (%)

N. B. appropriate conversion factors are applied to keep units consistent.

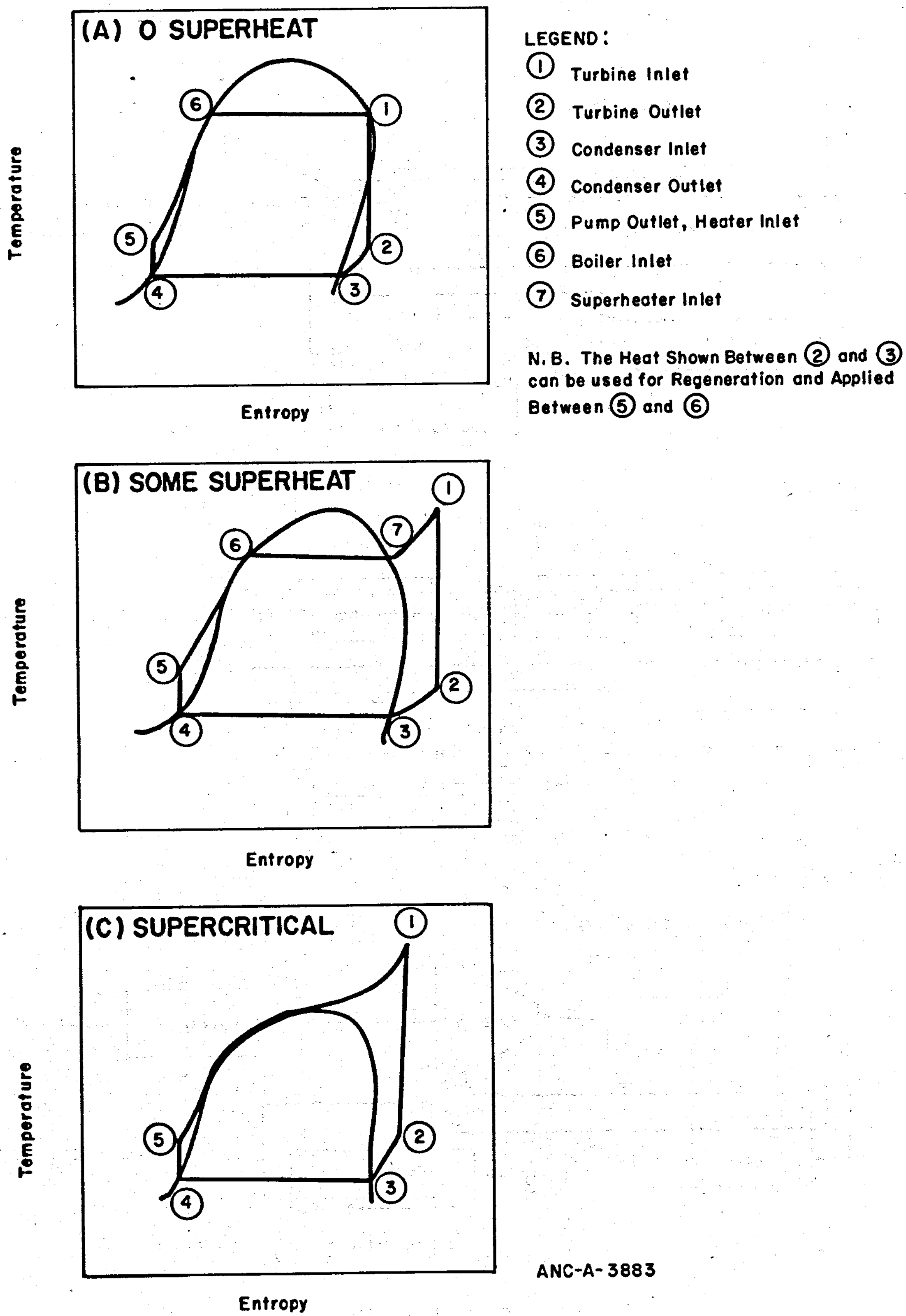

Fig. 1 Variations on rankine cycles.

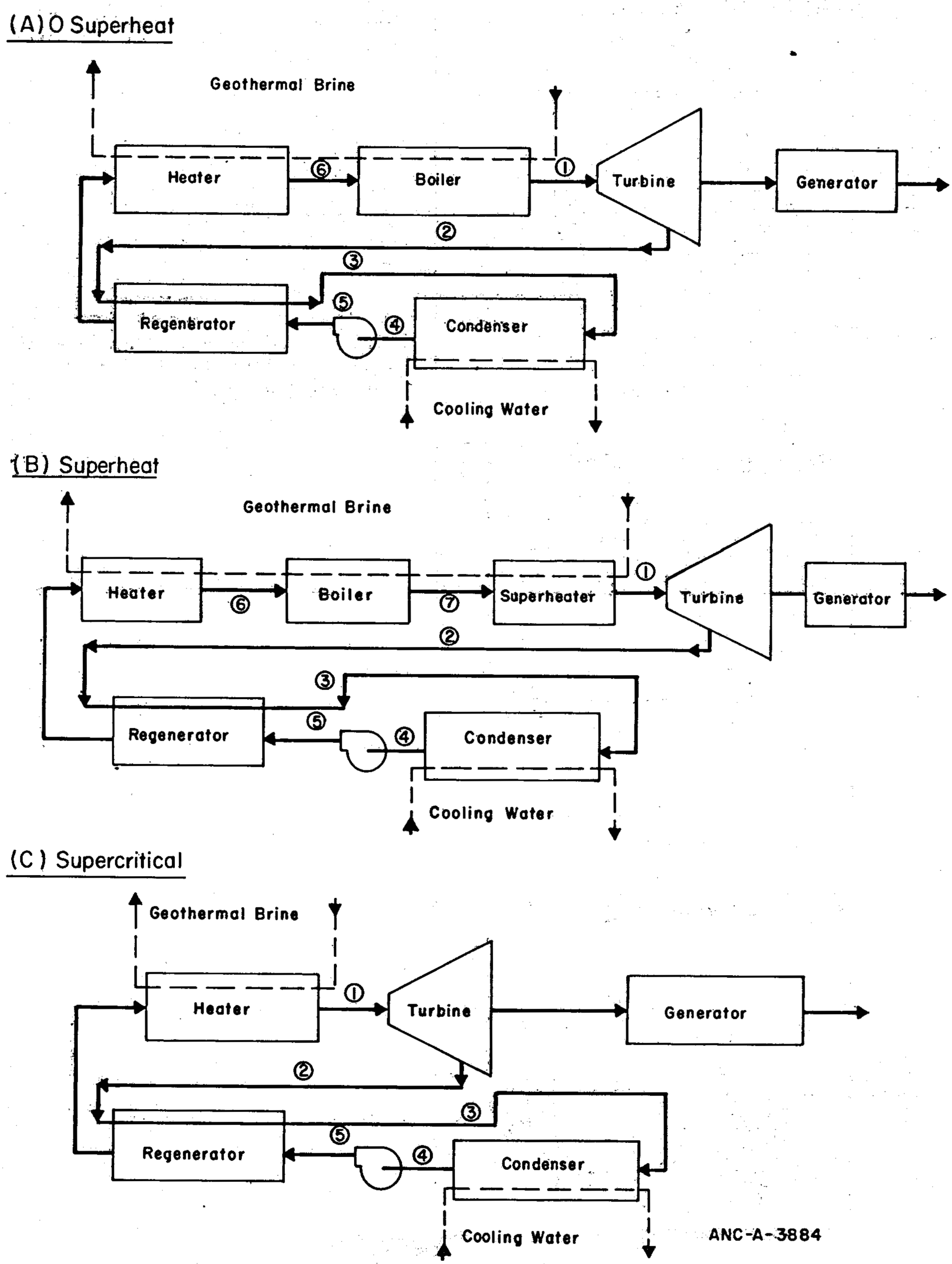

Fig. 2 Variations on rankine cycles.

2.2 The Equations

(1) Working fluid flow rate

$$\dot{w} = \frac{W_N}{C_{PG}\, T_{IN}\, \eta_t \left[1 - \left(\frac{P_{IN}}{P_{OUT}}\right)^{\frac{1-k}{K}}\right] - \left[\frac{v_L}{\eta_p}\left(P_{IN} - P_{OUT} + \Delta P_{Po} + \Delta P_{PI}\right)\right]} \quad \left(\frac{lb}{hr}\right) .$$

(2) Heat available for regeneration

$$\Delta h_R = C_{PG} \left\{ T_{IN} - T_{COND} - T_{IN}\, \eta_t \left[1 - \left(\frac{P_{IN}}{P_{OUT}}\right)^{\frac{1-k}{k}}\right]\right\} \quad \left(\frac{Btu}{lb}\right) .$$

(3) External heat input required

$$\Delta h_T = C_{PL}\,(T_B - T_{COND}) + \Delta h_B + C_{PG}\,(T_{IN} - T_B) - \Delta h_R \quad \left(\frac{Btu}{lb}\right) .$$

(4) Thermal efficiency

$$\eta_{et} = \frac{W_N}{\dot{W}\, \Delta h_T} \quad (\%) .$$

(5) Amount of superheat

$$\Delta T_S = \frac{\Delta h_B\,(T_{BI} - T_B) + \Delta T_P\, C_{PG}\,(T_{IN} - T_B)}{C_{PG}\,(T_{IN} - T_B) + \Delta h_B} \quad (^\circ F) .$$

N. B. ΔT_P, the pinch point, is a hardware-dependent parameter[a]. It controls the amount of heat which can be transferred from the brine at a given flow rate by putting a lower bound on the brine temperature at the boiler entrance. By defining ΔT_S as the difference between the brine temperature and the fluid temperature at the boiler outlet, and ΔT_P as that difference at the inlet, certain simplifications can be made. If $\Delta h_B = 0$ (supercritical), $\Delta T_S = \Delta T_P$. If $T_B = T_{IN}$ (no superheat), $\Delta T_S = T_{BI} - T_B$.

[a] A small ΔT_P indicates a large heat exchanger.

(6) Condensing water flow rate

$$\dot{w}_C = \frac{\dot{w}(\Delta h_{COND} + \Delta h_R)}{\Delta T_{COND}} \quad .$$

(7) Final or outlet brine temperature

(a) For the case of no superheat ($T_{IN} = T_B$)

$$T_{BO} = T_B + \Delta T_p - \frac{\left[C_{PL} \; (T_B - T_{COND}) - \Delta h_R \right] \left[\Delta T_S - \Delta T_P \right]}{\Delta h_B} \quad (°R)$$

(b) For the case of superheat ($T_{IN} > T_B$) and for the supercritical case (Δh_B = 0)

$$T_{BO} = T_B + \Delta T_P - \frac{\left[C_{PL} \; (T_B - T_{COND}) - \Delta h_R \right] \left[T_{BI} - T_B - \Delta T_S \right]}{C_{PG} \; (T_{IN} - T_B)} \quad (°R) \quad .$$

This equation is derived assuming that the pinch point, ΔT_p, controls the flow. In some cases, where the brine inlet temperature is sufficiently higher than the boiler temperature, the brine temperature does not need to drop the full distance to $T_B + \Delta T_P$ (at the boiler inlet) in order to provide the heat addition necessary. In this instance, the pinch point does not control the brine flow rate.

A new parameter, ΔT_2, is introduced. ΔT_2 is another hardware-based parameter which sets the temperature difference between the brine and the working fluid at the entrance to the heater. If there is a regenerator in the system, this point is at the regenerator outlet. If there is no regenerator, it is at the pump outlet at the condensing temperature. The temperature of the working fluid at this point is $T_{COND} + \frac{\Delta h_R}{C_{PL}}$, where $\frac{\Delta h_R}{C_{PL}}$ is the temperature rise across the regenerator. If there is no regenerator, this reduces to T_{COND}. Then, the temperature of the brine is the temperature of the working fluid + ΔT_2, or $T_{BO} = T_{COND} + \frac{\Delta h_R}{C_{PL}} + \Delta T_2$.

If the pinch point does not control, the use of the controlling equation will artificially set the brine temperature at a value which is too low. This can be checked if it is noted that the brine outlet temperature cannot be lower than the fluid temperature at the heater inlet, or $T_{BO} > T_{COND} + \frac{\Delta h_R}{C_{PL}}$. If this is not true, T_{BO} has been artificially lowered, and the pinch point does not control.

Therefore, T_{BO} must be calculated for each case of superheat cycle and then must be compared to $T_{COND} + \frac{\Delta h_R}{C_{PL}}$ to check for pinch point control. If $T_{BO} \leq T_{COND} + \frac{\Delta h_R}{C_{PL}}$, T_{BO} must be recalculated as $T_{BO} = T_{COND} + \frac{\Delta h_R}{C_{PL}} + \Delta T_2$. Since there is no boiler in a supercritical cycle, the pinch point is not involved, and the noncontrolling equation is used.

Then, Equation (7.b) is amended to:

(b) For the cases of superheat and supercritical

(1) Pinch point controlling

$$T_{BO} = T_B + \Delta T_P - \frac{\left[C_{PL}\ (T_B - T_{COND}) - \Delta h_R\right]\left[T_{BI} - T_B - \Delta T_S\right]}{C_{PG}\ (T_{IN} - T_B)}\ (^\circ R)$$

(2) Pinch point not controlling (including supercritical)

$$T_{BO} = T_{COND} + \frac{\Delta h_R}{C_{PL}} + \Delta T_2\ (^\circ R)$$

(8) Brine flow rate

(a) For the case of no superheat

$$\dot{w}_B = \frac{\dot{w}\ \Delta h_B}{\Delta T_S - \Delta T_P}\ (\frac{lb}{hr})$$

(b) For the cases of superheat and supercritical

(1) Pinch point controlling

$$\dot{w}_B = \frac{\dot{w}\ C_{PG}\ (T_{IN} - T_B)}{T_{BI} - T_B - \Delta T_S}\ (\frac{lb}{hr})$$

(2) Pinch point not controlling (including supercritical)

$$\dot{w}_B = \frac{\dot{w}\ \Delta h_T}{T_{BI} - T_{COND} - \frac{\Delta h_R}{C_{PL}} - \Delta T_2}\ (\frac{lb}{hr})\ .$$

(9) Effectiveness

$$E = \frac{W_N}{\dot{w}_B}\ (\frac{Btu}{lb\ of\ Brine})\ .$$

3. EVALUATION OF RESULTS

A program for the Wang 700 was set up to compute the parameters designated in Section IV-2. After reducing the results, it was found that the assumption that C_P does not vary significantly in the vapor region led to unacceptable values. (C_P in the liquid region was acceptably stable.) Therefore, the variation in C_{PG} must be accounted for. In lieu of writing a more complicated program for the IBM 360-75 which would account for the varying C_{PG}, it was decided that hand calculations using enthalpy values found in the literature would be the fastest approach. The equations in Section IV-2 are now modified as follows:

3.1 Definition of New Terms

N. B. All enthalpies refer to the working fluid.

h_{IN} The enthalpy at the turbine inlet (Btu/lb)

$h_{OUT})_S$ The enthalpy at the turbine outlet assuming isentropic expansion (Btu/lb)

h_{CL} The enthalpy at condenser temperature for the saturated liquid (Btu/lb)

h_{CG} The enthalpy at condenser temperature for the saturated vapor (Btu/lb)

h_{BL} The enthalpy at boiler temperature for the saturated liquid (Btu/lb)

h_{BG} The enthalpy at boiler temperature for the saturated vapor (Btu/lb).

3.2 Revised Equations

(1) Working fluid flow rate

$$\dot{w} = \frac{W_N}{\eta_{et}\left[(h_{IN} - h_{OUT})_s\right] - \frac{v_L}{\eta_P}(P_{IN} - P_{OUT} + \Delta P_{PO} + \Delta P_{PI})}\ (\frac{lb}{hr})$$

(2) Heat available for regeneration

$$\Delta h_R = h_{IN} - \eta_t \left[(h_{IN} - h_{OUT})_s \right] - h_{CG} \ (\frac{Btu}{lb}) \ .$$

(3) External heat input required

$$\Delta h_T = h_{IN} - h_{CL} - \Delta h_R \ (\frac{Btu}{lb}) \ .$$

(4) Thermal efficiency

$$\eta_{et} = \frac{W_N}{\dot{w} \ \Delta h_T} \ (\%) \ .$$

(5) Amount of superheat

$$\Delta T_S = \frac{\Delta h_B \ (T_{IN} - T_B) + \Delta T_P \ (h_{IN} - h_{BG}) \ (°F)}{\Delta h_B + (h_{IN} - h_{BG})} \ .$$

(6) Condensing water flow rate

$$\dot{w}c = \frac{\dot{w} \ (\Delta h_{COND} + \Delta h_R)}{T_{COND}} \ (\frac{lb}{hr}) \ .$$

(7) Final or outlet brine temperature

(a) For the case of no superheat ($T_{IN} = T_B$)

$$T_{BO} = T_B + \Delta T_P - \frac{(h_{BL} - h_{CL} - \Delta h_R) \ (\Delta T_S - \Delta T_P)}{\Delta h_B} \quad (°R)$$

(b) For the cases of superheat ($T_{IN} < T_B$) and supercritical ($\Delta h_B = 0$)

(1) Pinch point controlling

$$T_{BO} = T_B + \Delta T_P - \frac{(h_{BL} - h_{CL} - \Delta h_R)\ (T_{BI} - T_B - \Delta T_S)}{h_{IN} - h_{BG}} \quad (°R)$$

(2) Pinch point not controlling (including supercritical)

$$T_{BO} = T_{COND} + \frac{\Delta h_R}{C_{PL}} + \Delta T_2 \quad (°R) \ .$$

(8) Brine flow rate

(a) For the case of no superheat ($T_{IN} = T_B$)

$$\dot{w}_B = \frac{\dot{w}\ \Delta h_B}{\Delta T_S - \Delta T_P} \quad (\frac{lb}{hr})$$

(b) For the cases of superheat ($T_{IN} < T_B$) and supercritical ($\Delta h_B = 0$)

(1) Pinch point controlling

$$\dot{w}_B = \frac{\dot{w}\ (h_{IN} - h_{BG})}{T_{BI} - T_B - \Delta T_S} \quad (\frac{lb}{hr})$$

(2) Pinch point not controlling (including supercritical)

$$\dot{w}_B = \frac{\dot{w}\ \Delta h_T}{T_{BI} - T_{COND} - \frac{\Delta h_R}{C_{PL}} + \Delta T_2} \quad (\frac{lb}{hr}) \ .$$

(9) Effectiveness

$$E = \frac{W_N}{\dot{w}_B} \quad (\frac{Btu}{lb\ of\ brine}) \ .$$

4. INPUT PARAMETERS

These values were used for all cases. These are system parameters and vary according to hardware design.

W_N	=	3.79×10^7 Btu/hr		
ΔP_{PI}	=	10 psid		
ΔP_{PO}	=	5 psid		
T_{BI}	=	300^oF	=	760^oR
T_{COND}	=	80^oF	=	540^oR
T_{IN}	=	280^oF	=	740^oR
ΔT_P	=	10^o		
ΔT_2	=	20^o		
η_P	=	80%		
η_t	=	85% except for condensing turbines when $\eta_t = 75\%$.		

For the fluids certain physical properties were inputted.

	Isobutane	F-12	F-113
C_{PL} $(\frac{Btu}{lb-°R})$	0.70	0.32	0.25
Δh_{COND} $(\frac{Btu}{lb})$	140.5	58.92	65.88
k	1.11	1.14	1.08
P_{OUT} (Psia)	53	100	7
v_L	0.0300	0.0124	0.0102

The other input values vary with boiler temperature and must be designated for individual cases. For instance, ΔT_{COND} is taken as the turbine outlet temperature minus condensing temperature (80^oF) minus 10^o terminal difference for the heat exchanger. A maximum value of ΔT_{COND} is taken as 30^o. Both of these values (30^o and 50^o) are chosen as sample heat exchanger operating parameters.

V. RESULTS AND DISCUSSION

The results are shown in graphical form for the case of 0 superheat (Figures 3a, b to 7a, b) and for the superheated case (Figures 8a, b to 12a, b). The results for the critical and supercritical case are found tabulated in Table V. Figures 13 through 15 show working fluid flow rate, thermal efficiency, and effectiveness for varying turbine inlet temperature, at constant boiler temperature and pressure.

1. THE CASE OF 0 SUPERHEAT

The working fluid flow rate curves (Figures 3a, and 3b) show that generally, flow rate decreases as boiler temperature and pressure increase. Figure 3a shows the isobutane and F-12 curves bending back at higher temperatures. This is due to the corresponding higher pressures and the pump work losses. There is a breakeven point after which the pump work required is greater than the power gained (from raising boiler temperature), and a net loss results.

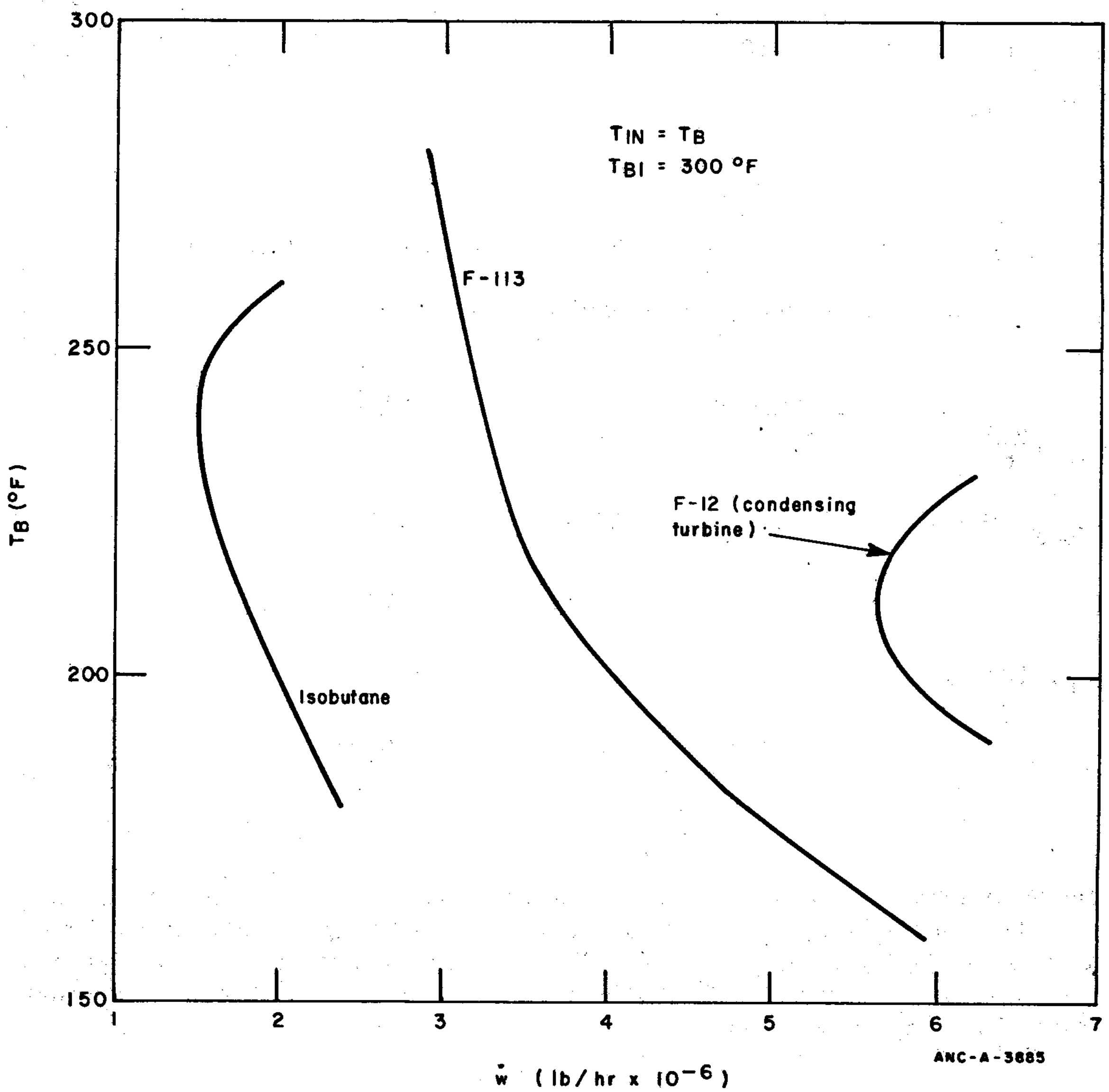

Fig. 3a O superheat cycle working fluid flow rate versus boiler temperature.

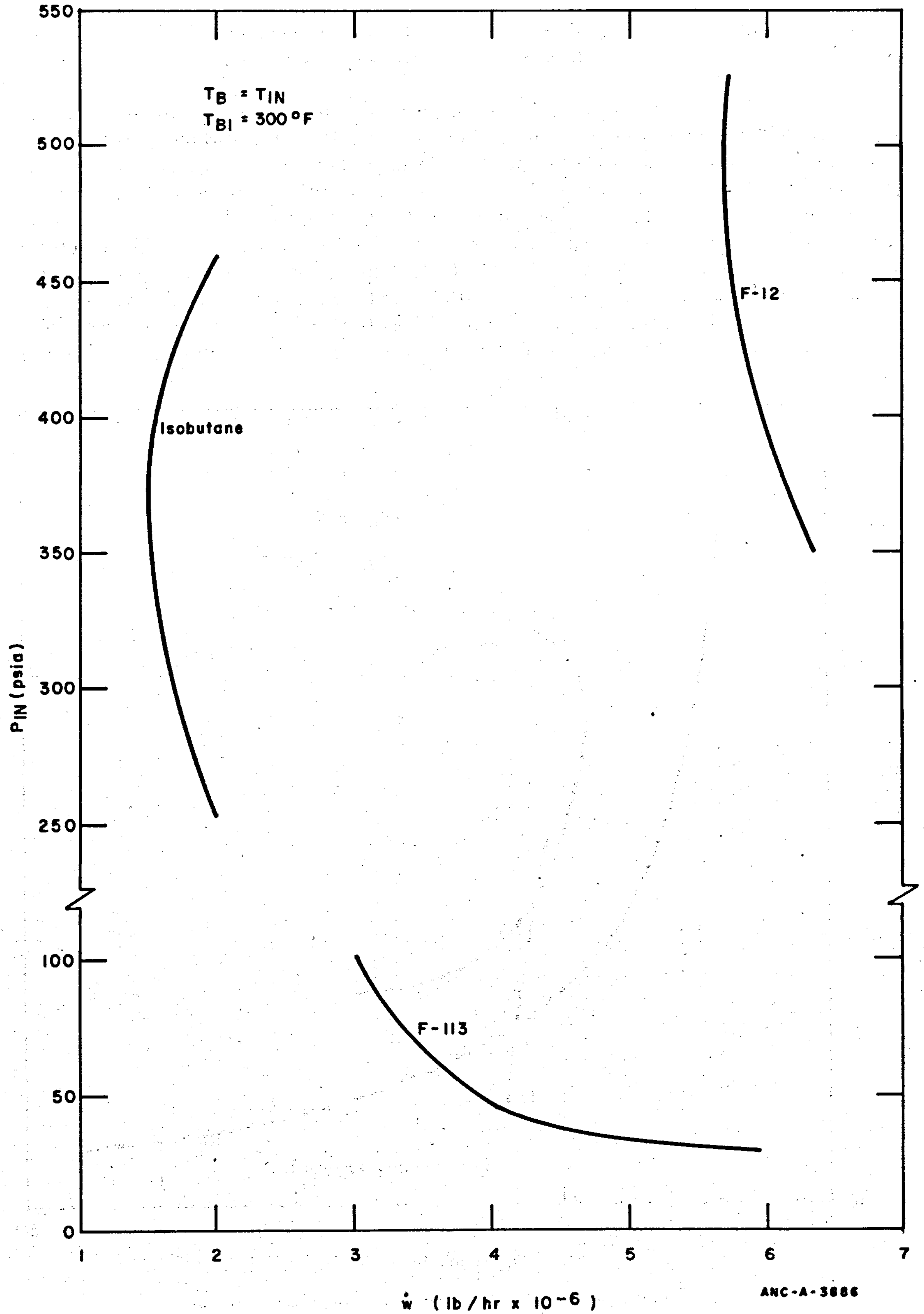

Fig. 3b O superheat cycle working fluid flow rate versus turbine inlet pressure.

In addition, the working fluid flow rate curves have a direct effect on condensing water flow rate and brine flow rate as well as upon the effectiveness curves.

The condensing water flow rate curves (Figures 4a, 4b) show a decrease in condensing water requirement as the boiler temperature and therefore, the turbine pressure ratio, increases. F-113 and isobutane both operate with turbines in the vapor region and have to account for the amount of working fluid used for regeneration that has to be condensed. The regenerator reduces coolant flow to a certain extent because the temperature of the working fluid is lower when it reaches the condenser, but the net effect is still an increase. The amount of regenerative heat increases as boiler temperature decreases, requiring greater cooling water flow. F-12 uses a condensing turbine, and the condensing water flow decreases as the turbine exhaust gets progressively wetter. This offsets the increase due to lower boiler temperatures, and the effect is to bend the curve backwards. This is more clearly shown in Figure 4b.

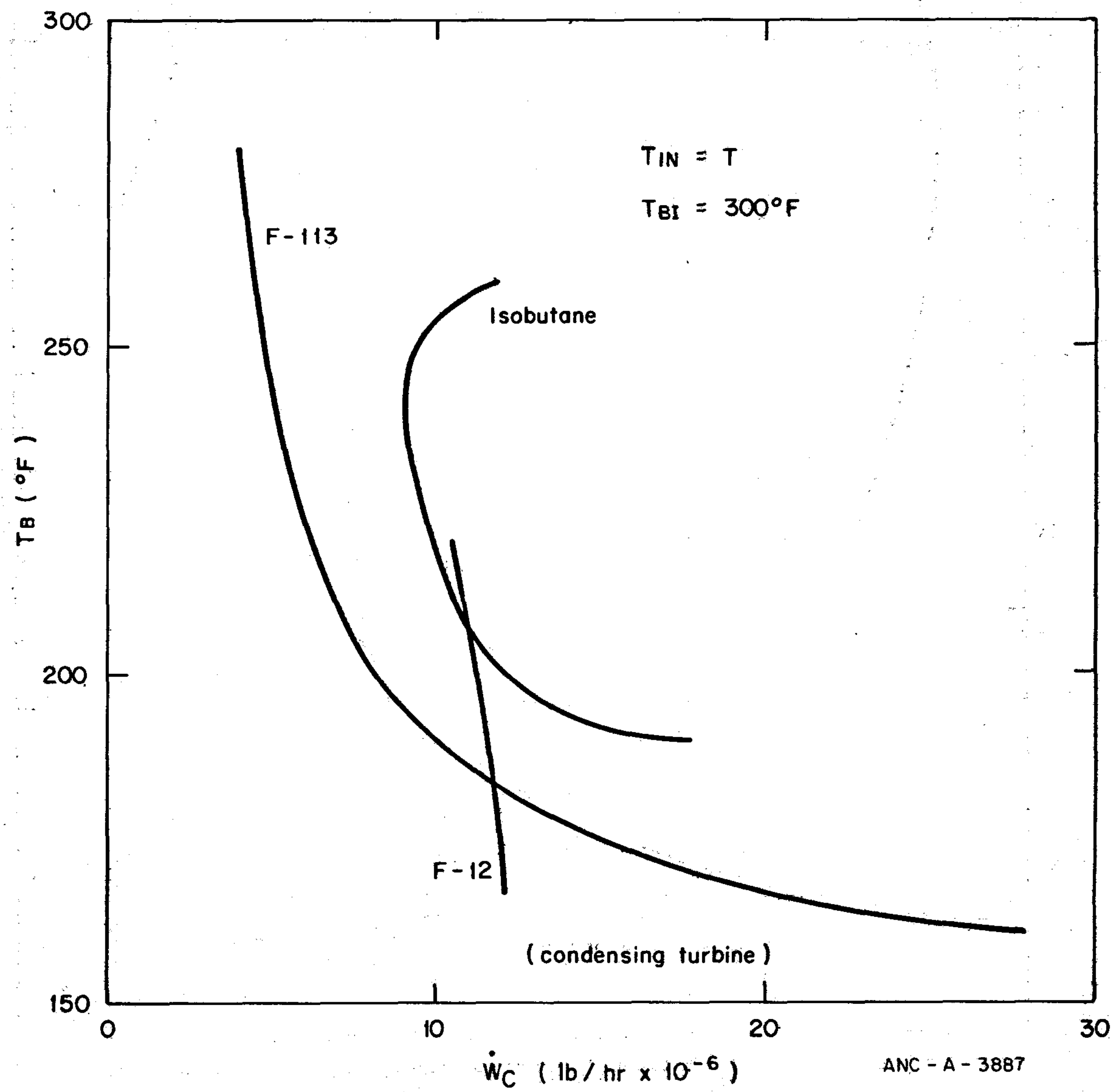

Fig. 4a O superheat cycle condensing water flow rate versus boiler temperature.

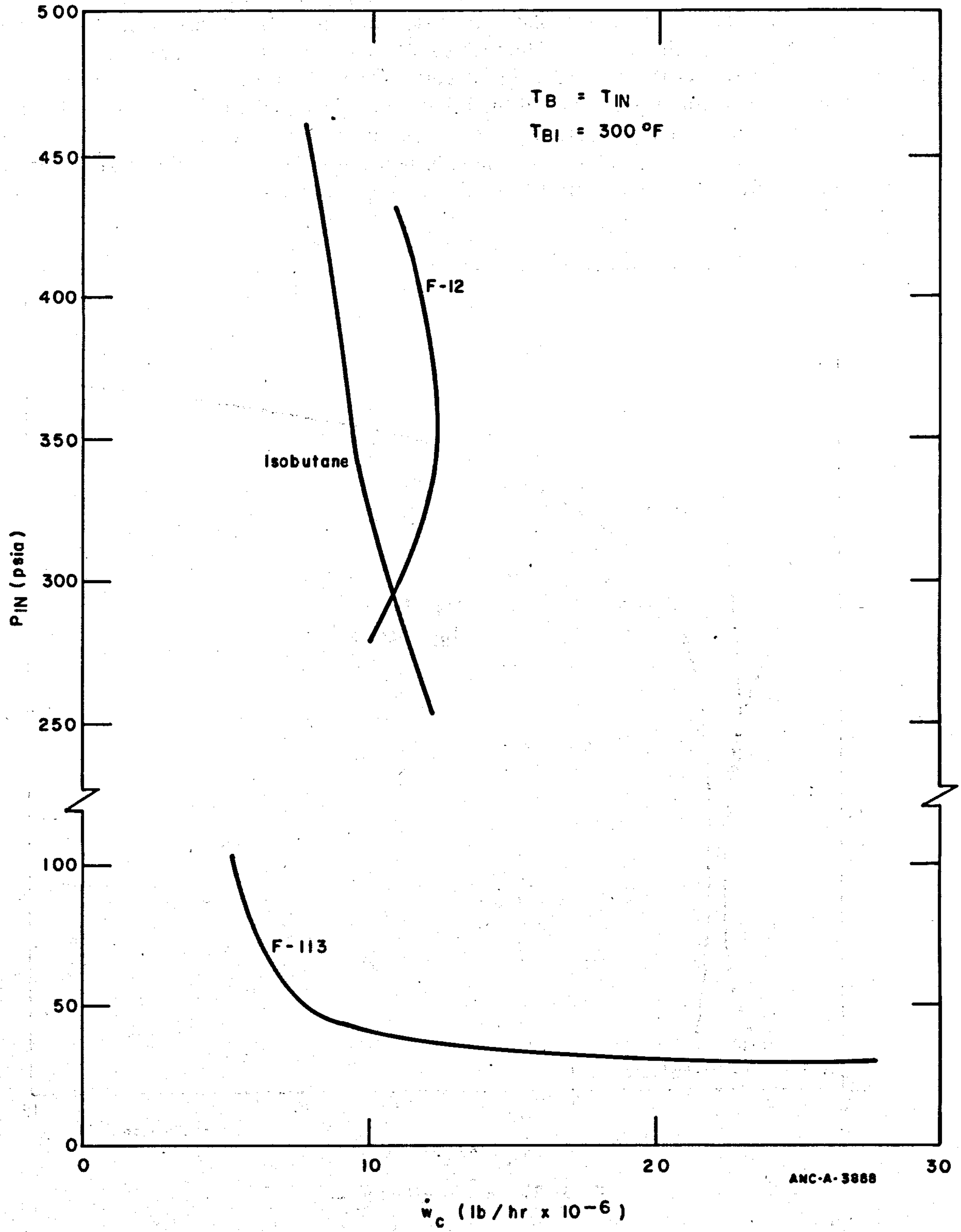

Fig. 4b 0 superheat cycle condensing water flow rate versus turbine inlet pressure.

Brine flow rate curves (Figures 5a, 5b) for F-113 and isobutane follow expected trends, reaching a minimum as boiler temperature increases, then increasing as the pump losses at higher pressure come into play. The F-12 curve has reverse curvature due to the influence of the condensing turbine. The turbine efficiency is taken as a constant 75%. This is not the true case. As the turbine exhaust gets wetter, the efficiency of the turbine at higher temperatures would decrease and the brine would decrease and follow the F-113 and isobutane curves. Boiler temperature has little effect upon brine flow rate (isobutane is also essentially linear, as is F-113 at lower temperatures) until the pinch point takes over at higher temperatures and causes a larger brine flow rate requirement.

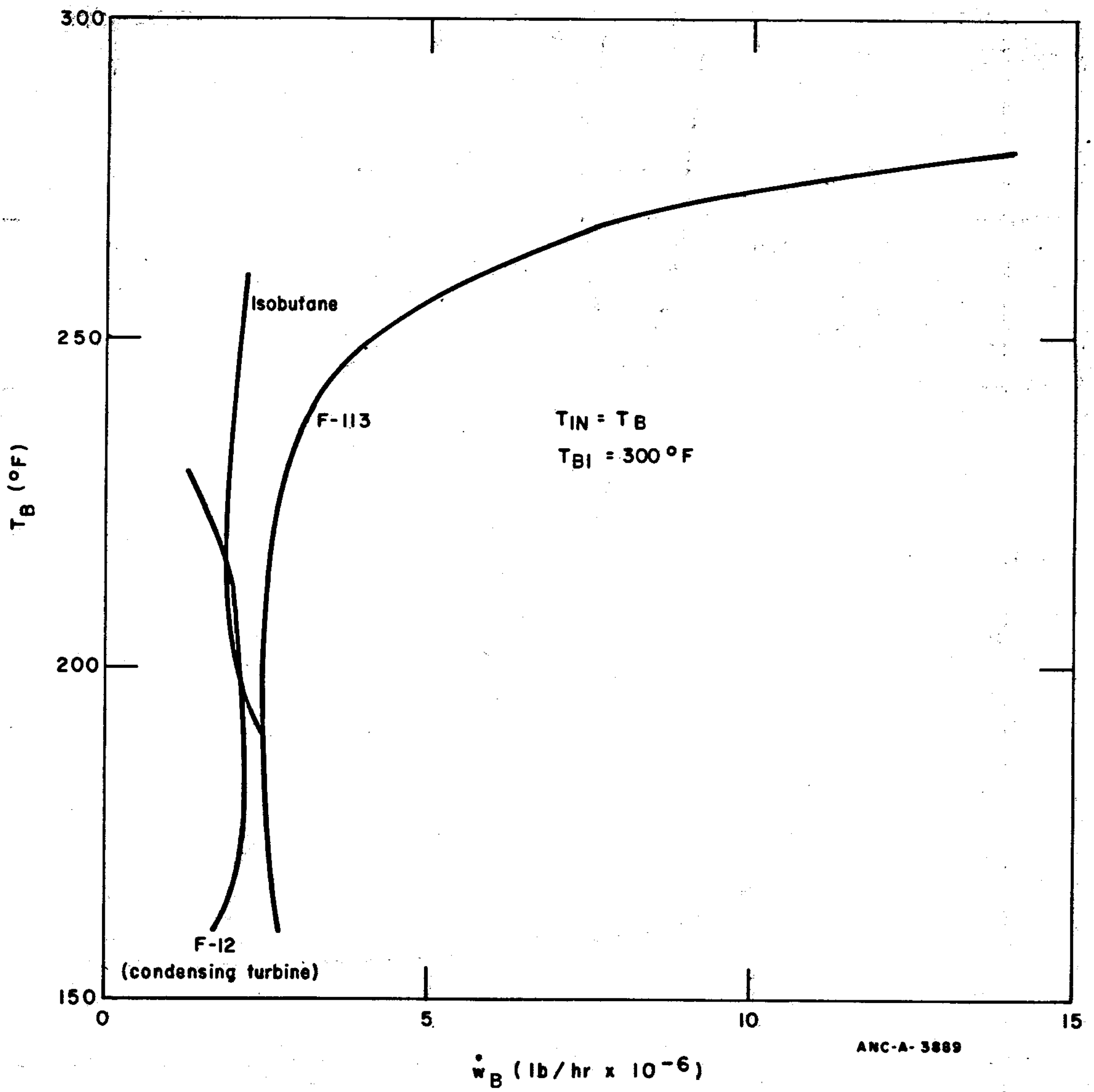

Fig. 5a Rankine cycle -- 0 superheat.

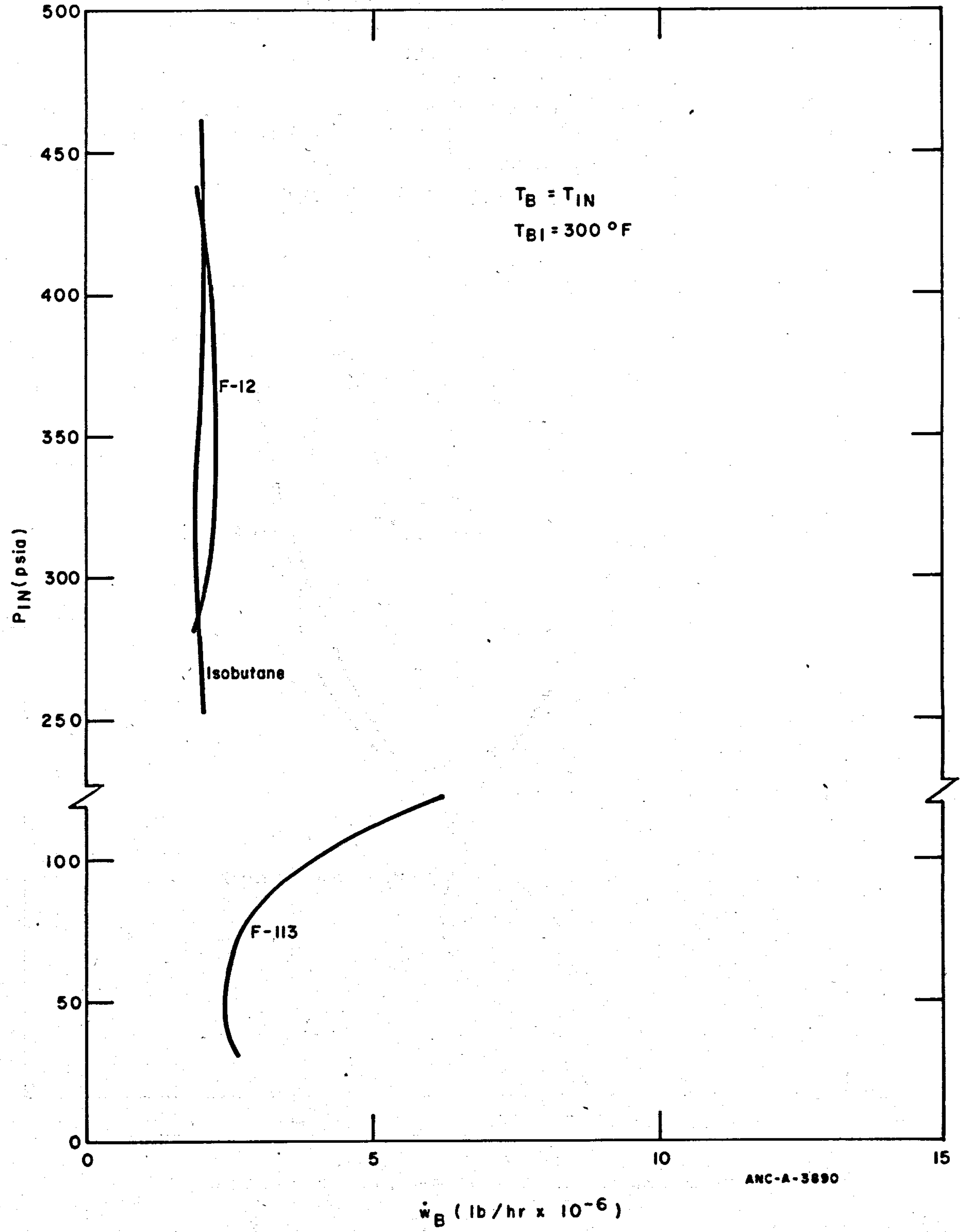

Fig. 5b O superheat cycle brine flow rate versus turbine inlet pressure.

Thermal efficiency curves (Figures 6a, 6b) show an increase in efficiency as lower temperatures (and pressures) increase, with the pump work losses bending back the curves at higher pressures. F-113 does not bend back as severely as the others because it is a lower pressure cycle and pumping losses are relatively small.

The effectiveness curves (Figures 7a, 7b) are inversely proportional to the brine flow rate curves. The F-12 curve clearly shows the effect of the condensing turbine. The F-113 and isobutane curves show a maximum effectiveness at some boiler temperature and pressure at which it would be most desirable to operate the cycles.

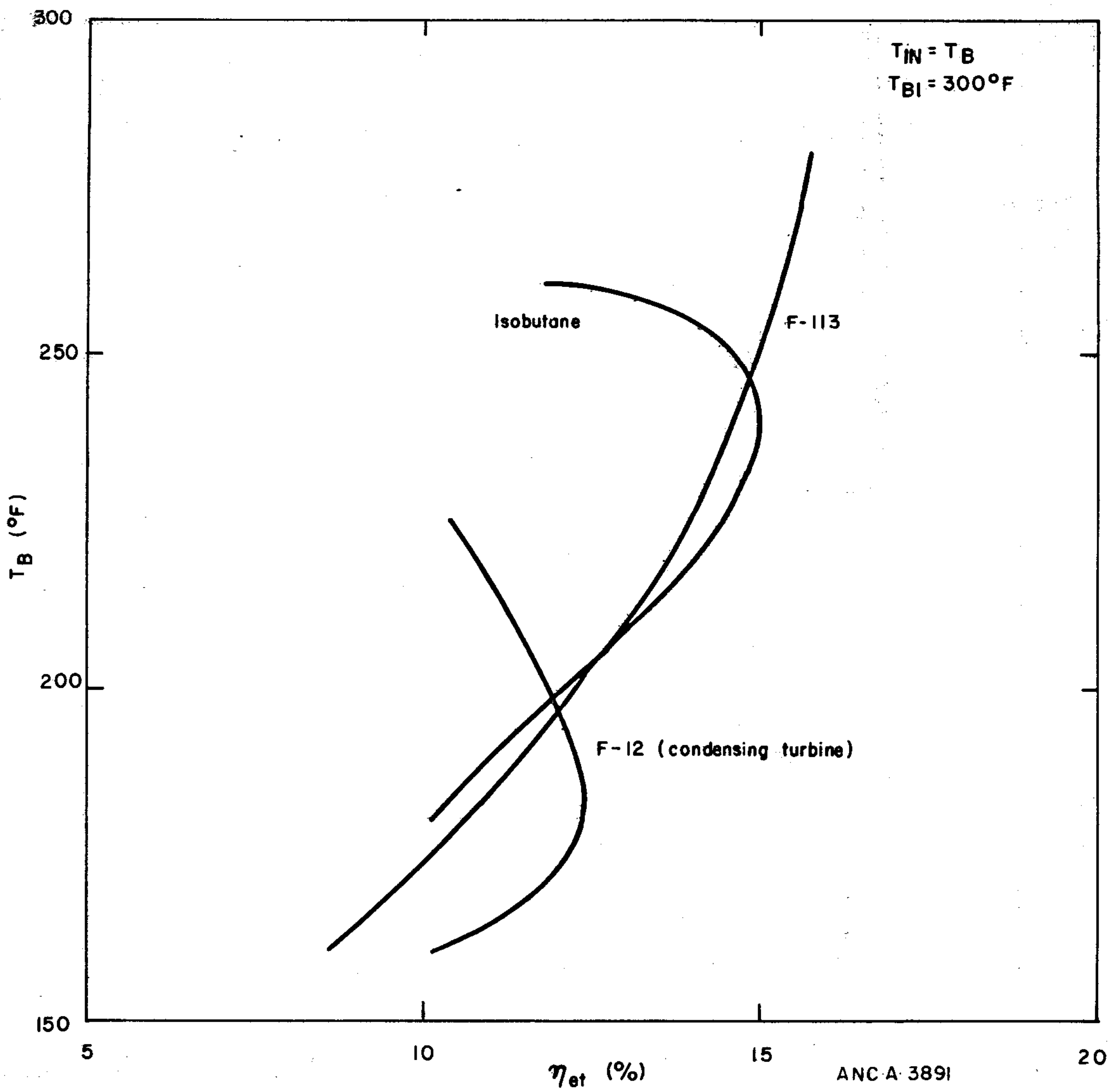

Fig. 6a 0 superheat cycle thermal efficiency versus boiler temperature.

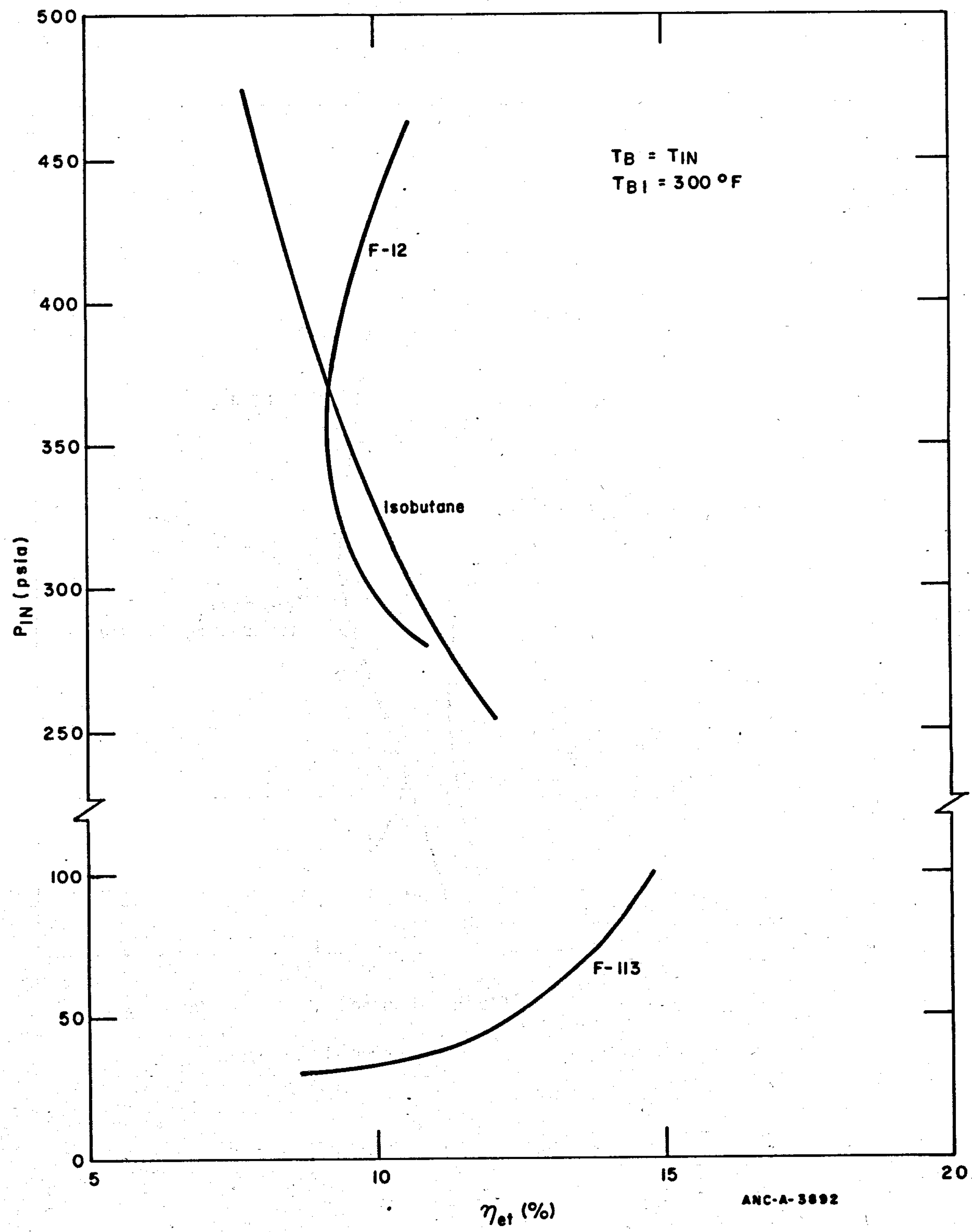

Fig. 6b O superheat cycle thermal efficiency versus turbine inlet pressure.

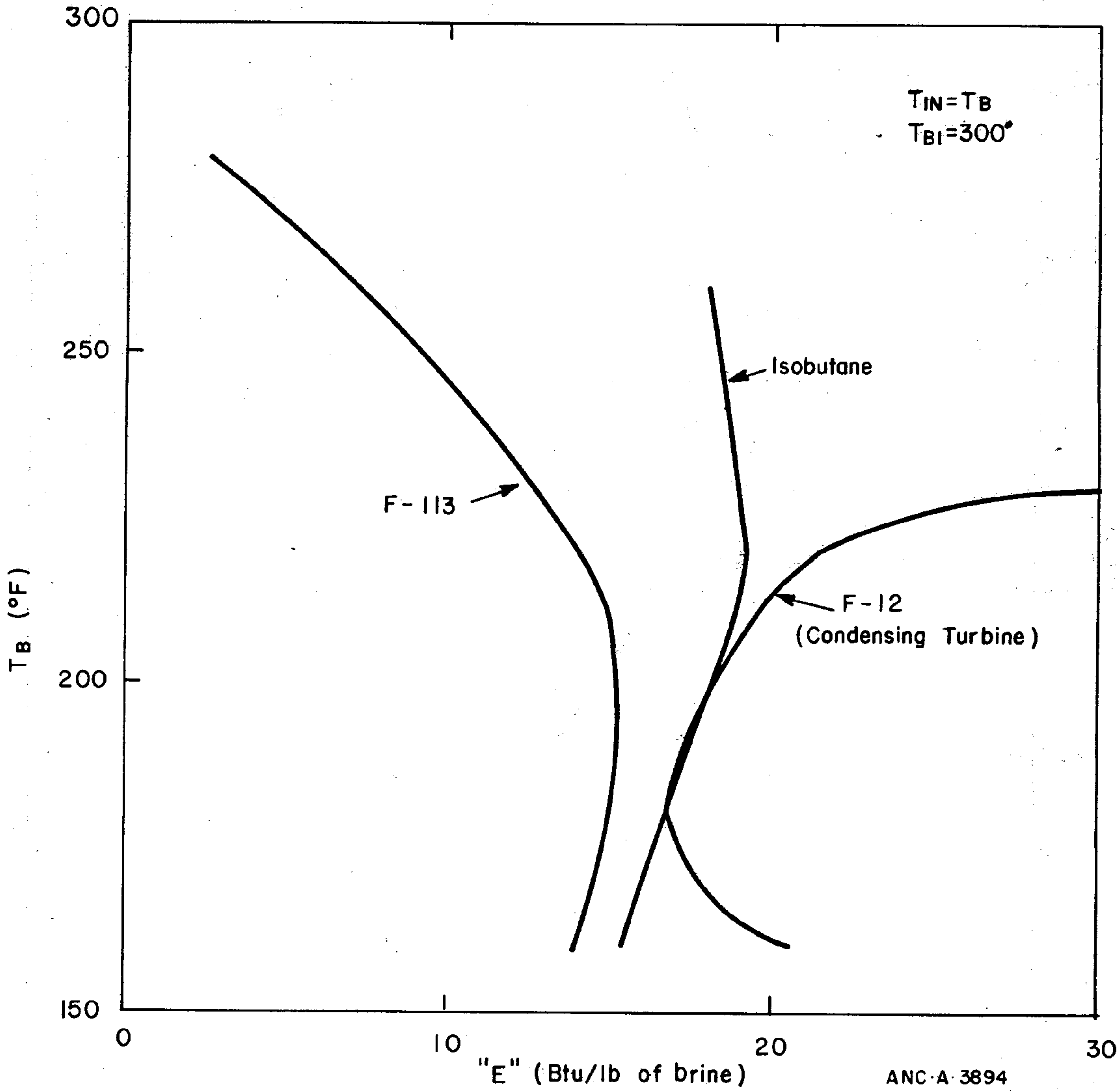

Fig. 7a 0 superheat cycle effectiveness versus boiler temperature.

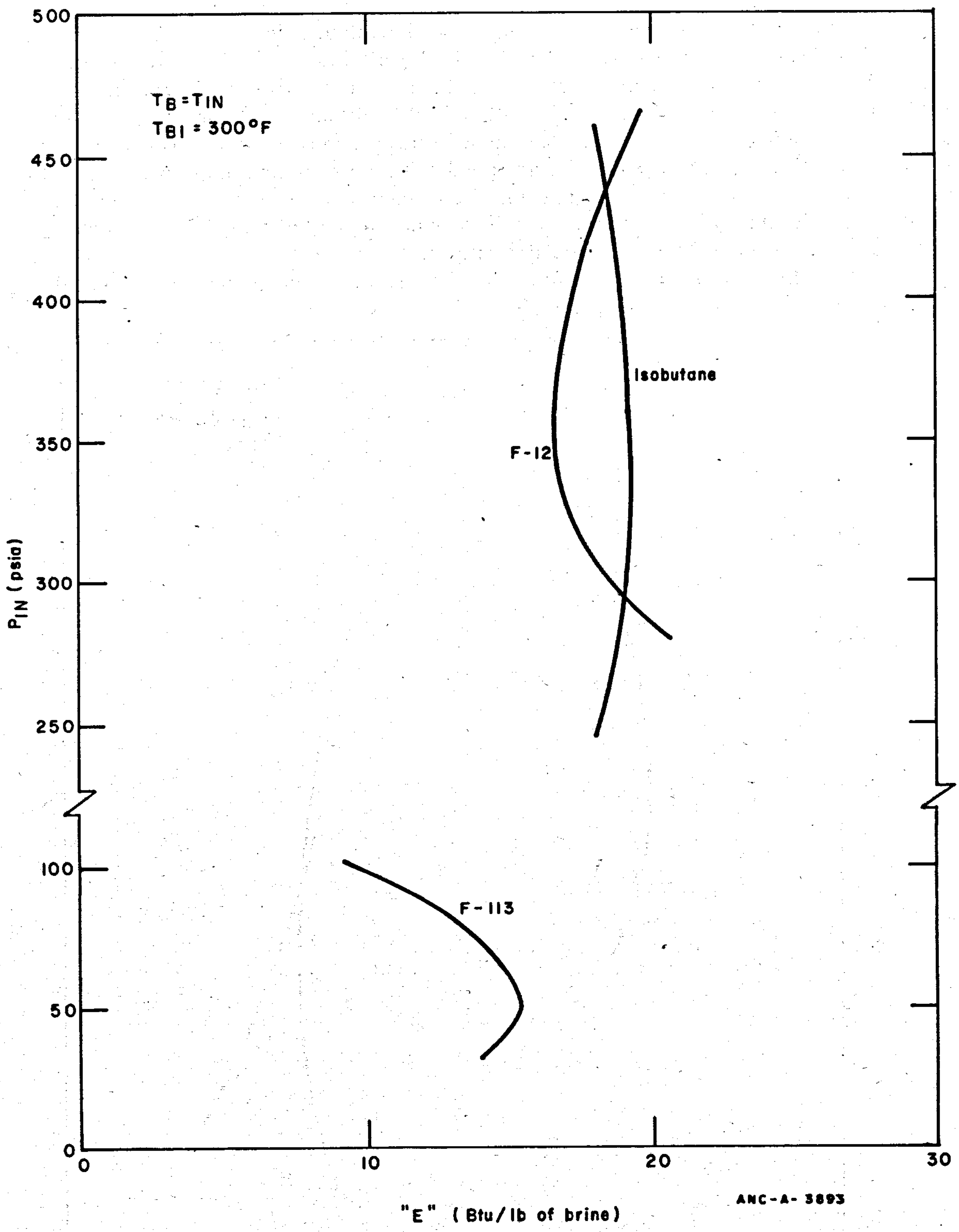

Fig. 7b 0 superheat cycle effectiveness versus turbine inlet pressure.

2. THE CASE OF SUPERHEATED WORKING FLUID

There is an interesting anomaly here in the working fluid flow rate curves (Figures 8a, 8b). F-12 and isobutane follow expected paths with higher flow rates at lower temperatures. F-113, however, shows a maximum flow rate, then bends backward. The nature of the F-113 pressure-enthalpy diagram is such that, at 280°F turbine inlet temperature, the inlet enthalpy corresponding to various boiler temperatures is almost constant. This makes the turbine output very nearly linear, and correspondingly, the working fluid flow rate. Due to the curvature of the saturated vapor line and the inclination of the constant entropy lines, slightly more turbine work is obtained at the extremes of the temperature range involved. Therefore, the working fluid flow rate decreases at these points. Or, in other words, the working fluid flow rate is not a strong function of pressure. This phenomenon is reflected in the other flow rates and efficiency parameters.

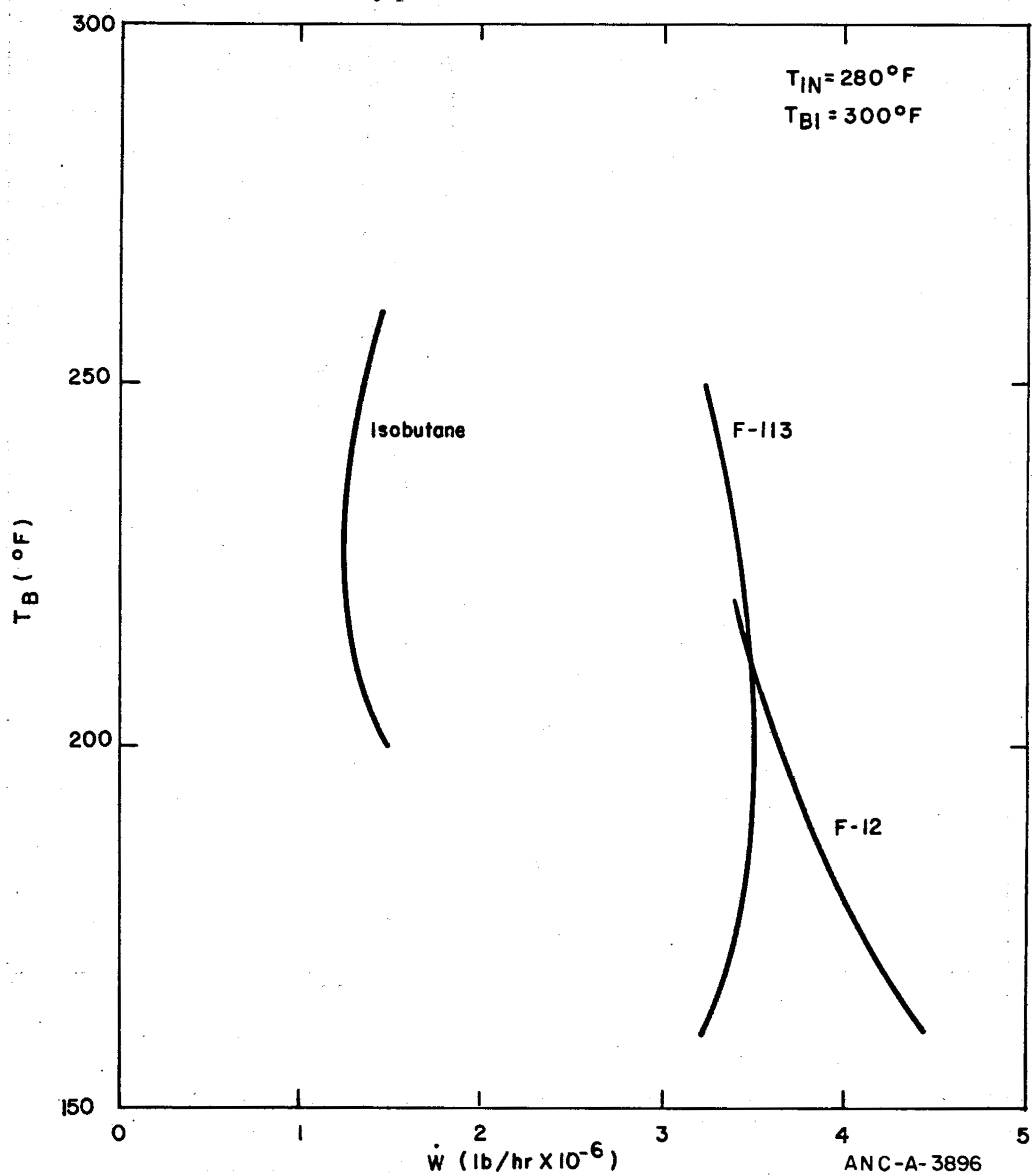

Fig. 8a Superheat cycle working fluid flow rate versus boiler temperature.

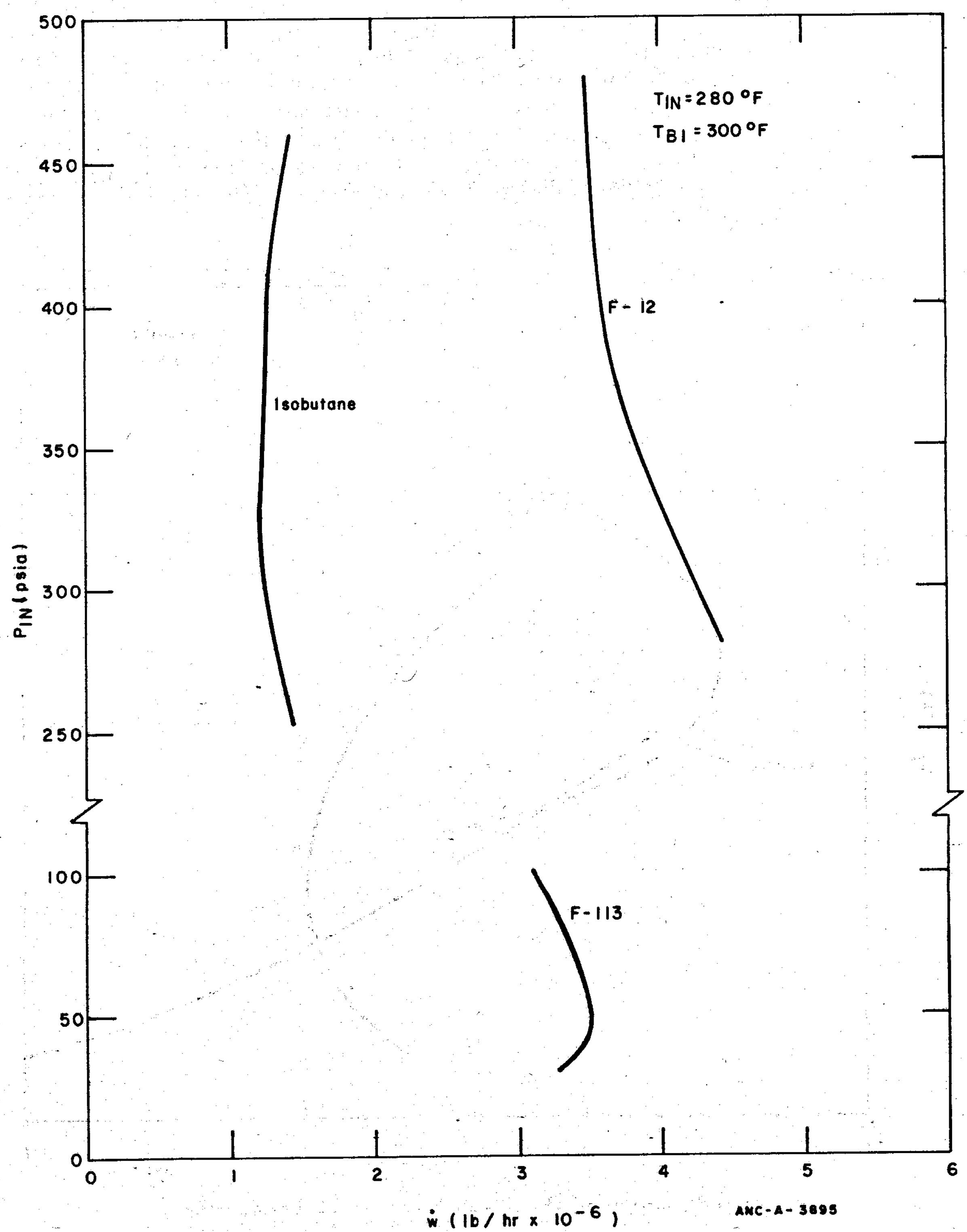

Fig. 8b Superheat cycle working fluid flow rate versus turbine inlet pressure.

The condensing fluid flow rate curves (Figures 9a, 9b) show a similarity to the working fluid flow rate curves. The isobutane curve bends back at higher temperatures because the 50^{o} ΔT_{COND} limiting case is used. If the ΔT_{COND} were set equal to the temperature of the turbine exhaust minus T_{COND} minus 10^{o} terminal difference, the curve would straighten out and the flow rate would decrease to approach a minimum value. This is also true for the F-12 case, but the working fluid flow rate curve for F-12 is nearly linear, and the effect of ΔT_{COND} is to round it out. In the case of F-113, the 50^{o} ΔT_{COND} controls, but the effect is more pronounced since the turbine exhaust minus T_{COND} minus 10^{o} terminal difference is in a higher range. This serves to accentuate the backward-bending effect of the working fluid flow rate curve.

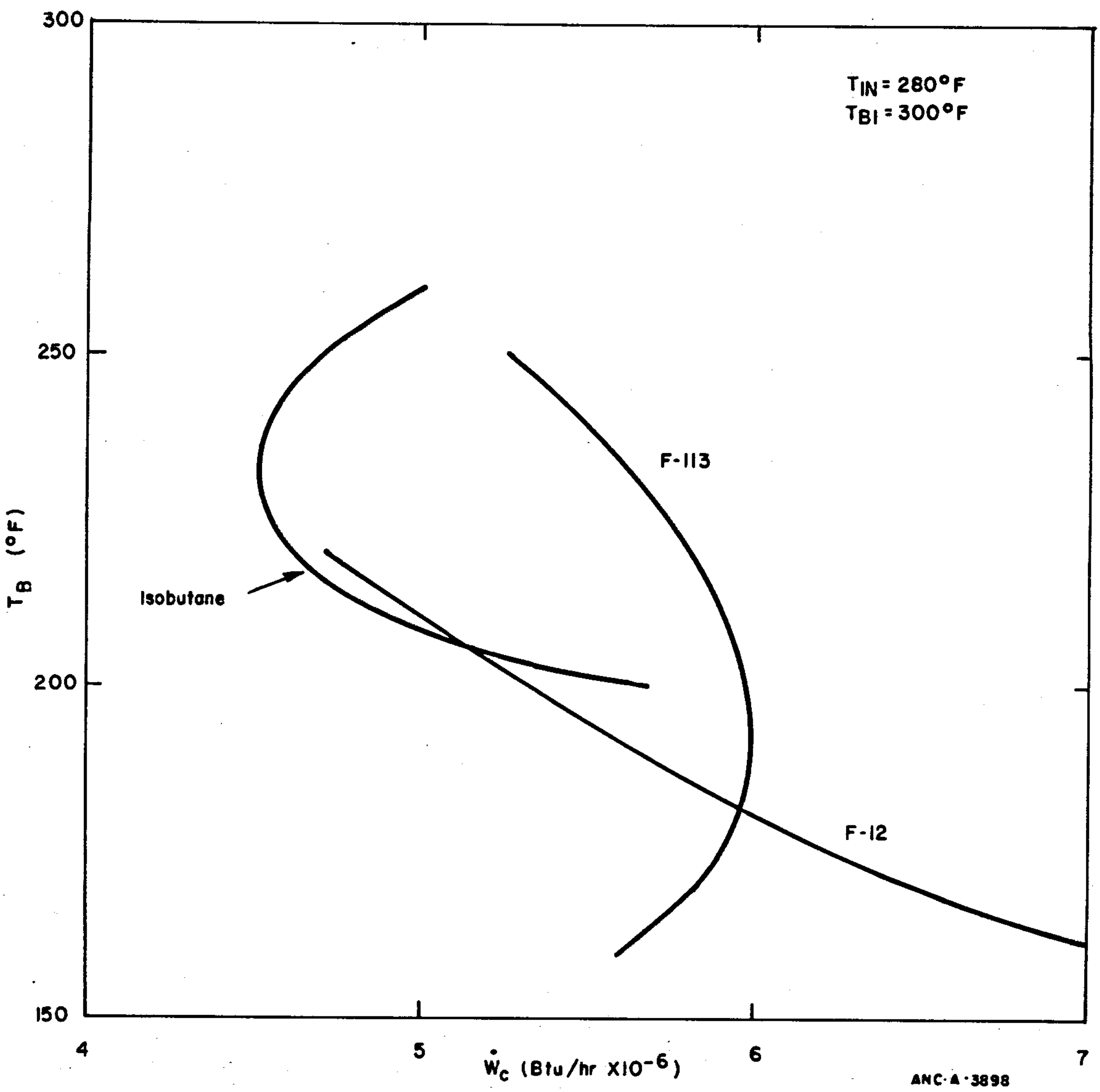

Fig. 9a Superheat cycle condensing water flow rate versus boiler temperature.

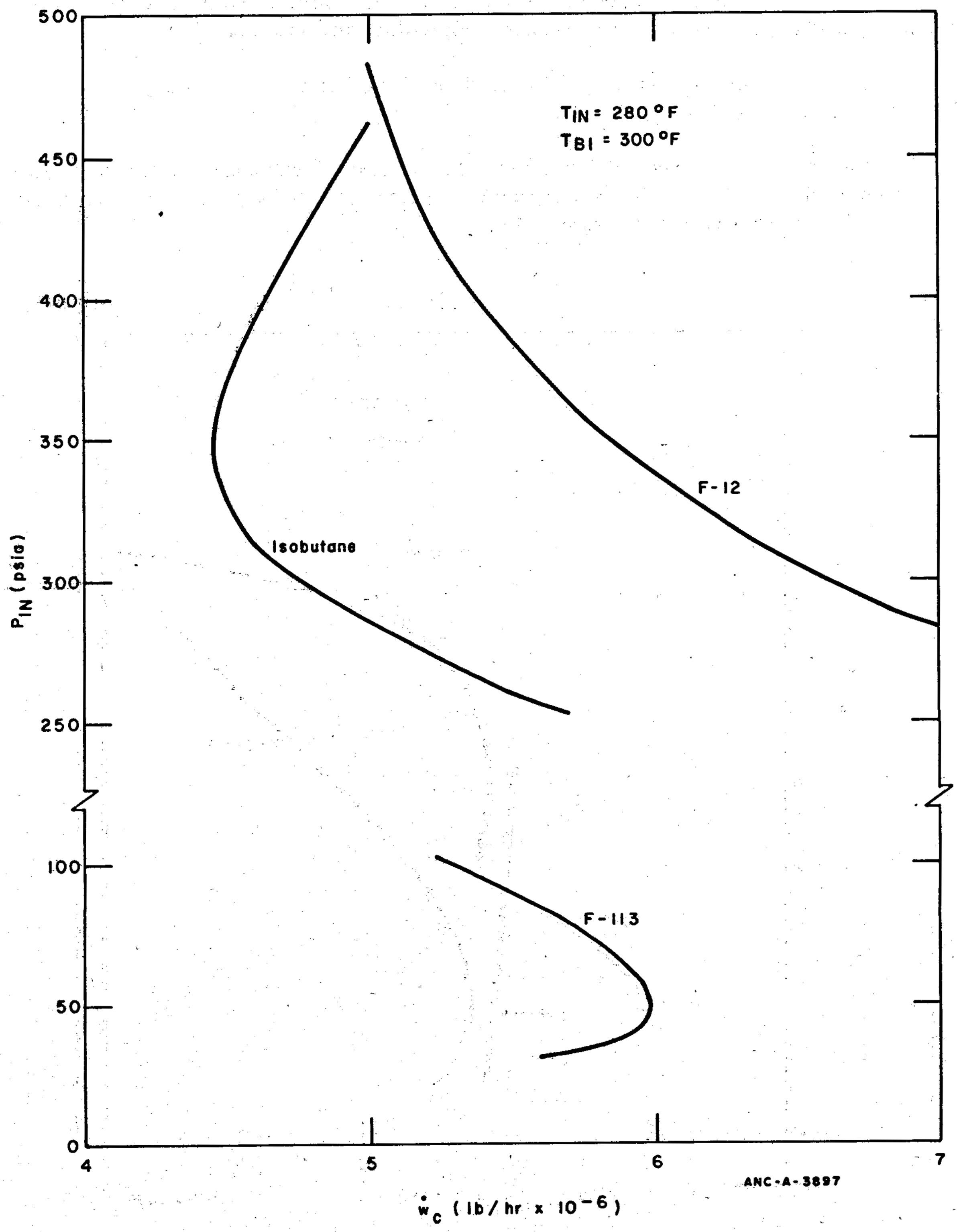

Fig. 9b Superheat cycle condensing water flow rate versus turbine inlet pressure.

The brine flow rate curves (Figures 10a, 10b) are fairly conventional. Again, F-12 exhibits its dependence on the nearly linear working fluid flow rate curve.

Thermal efficiency curves (Figures 11a, 11b) reiterate previously mentioned characteristics of the fluids.

The effectiveness curves (Figures 12a, 12b) are also self-explanatory. It is interesting to note in Figure 12b, that F-12 and isobutane operate at essentially the same pressures, but that F-113 is a relatively low pressure cycle.

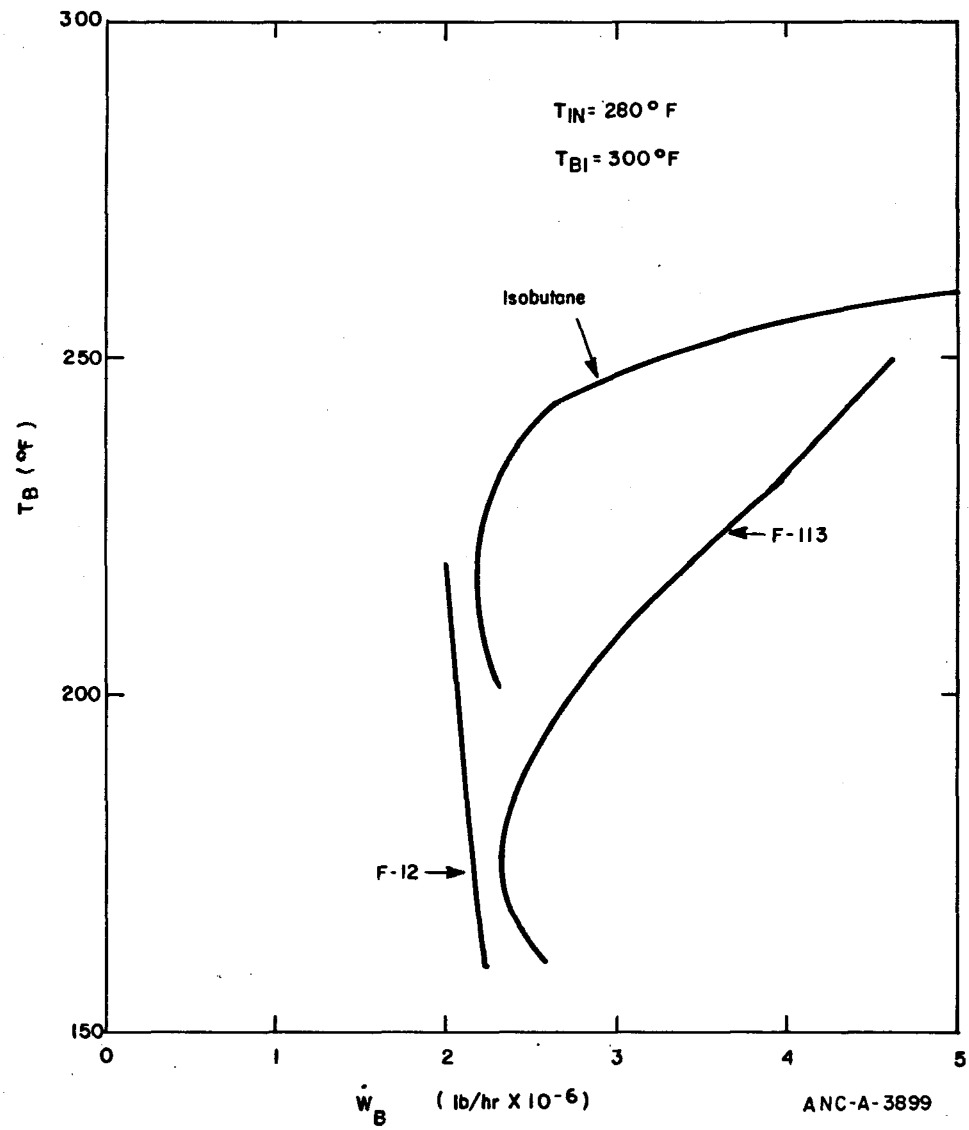

Fig. 10a Superheat cycle brine flow rate versus boiler temperature.

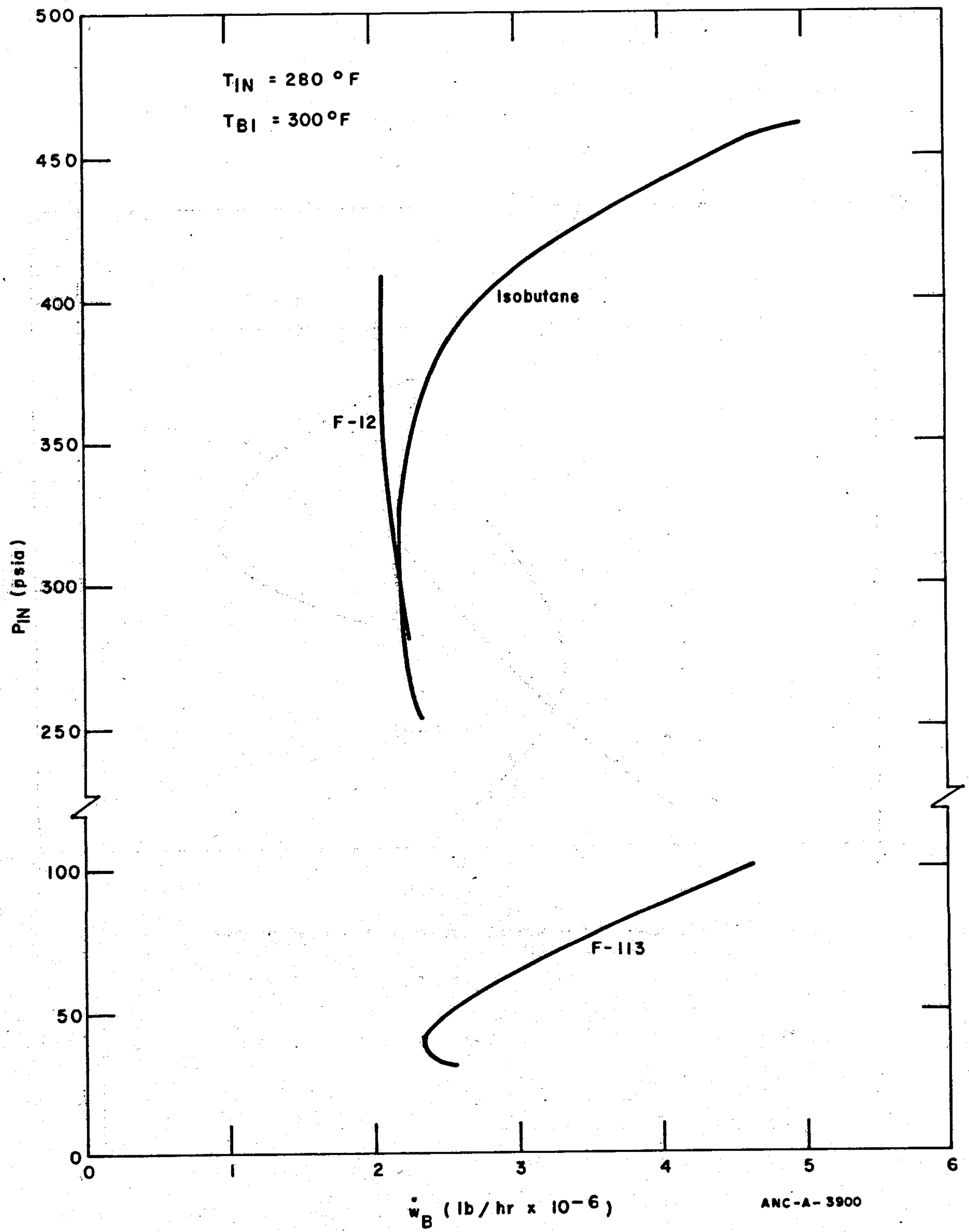

Fig. 10b Superheat cycle brine flow rate versus turbine inlet pressure.

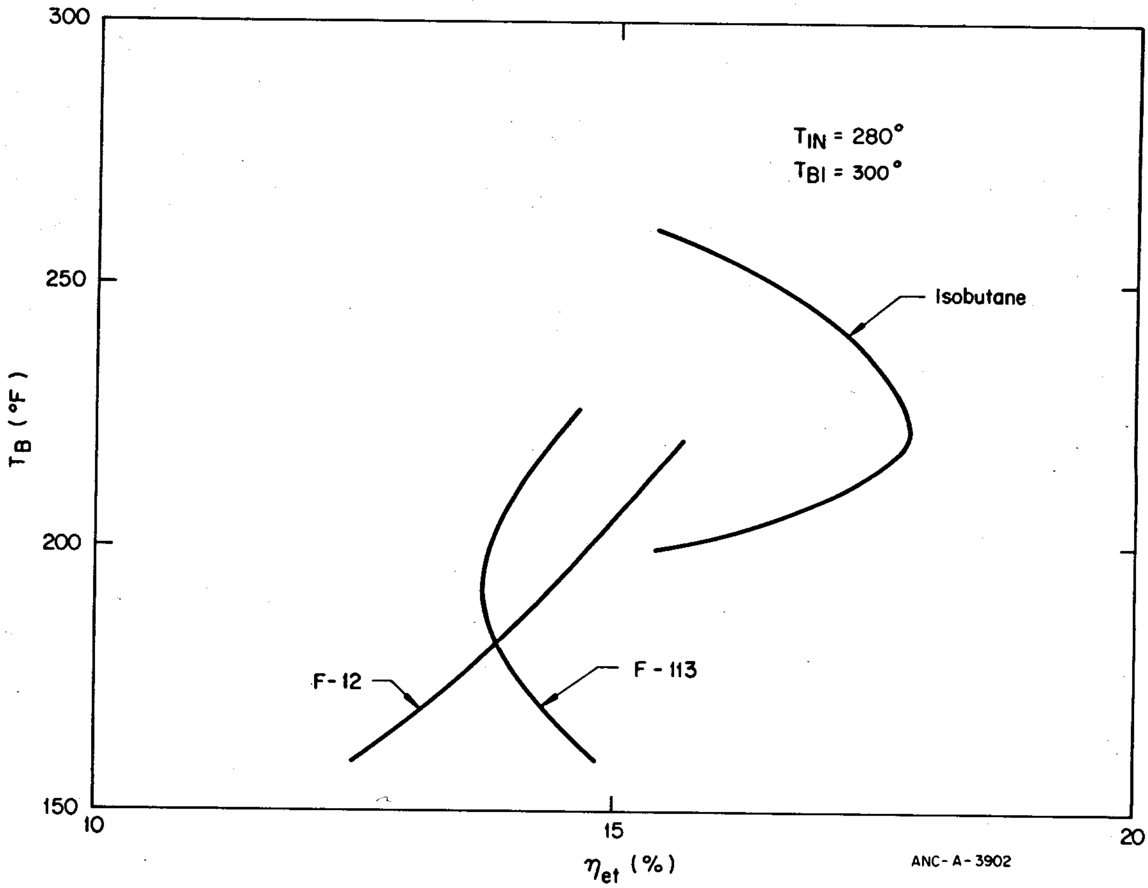

Fig. 11a Superheat cycle thermal efficiency versus boiler temperature.

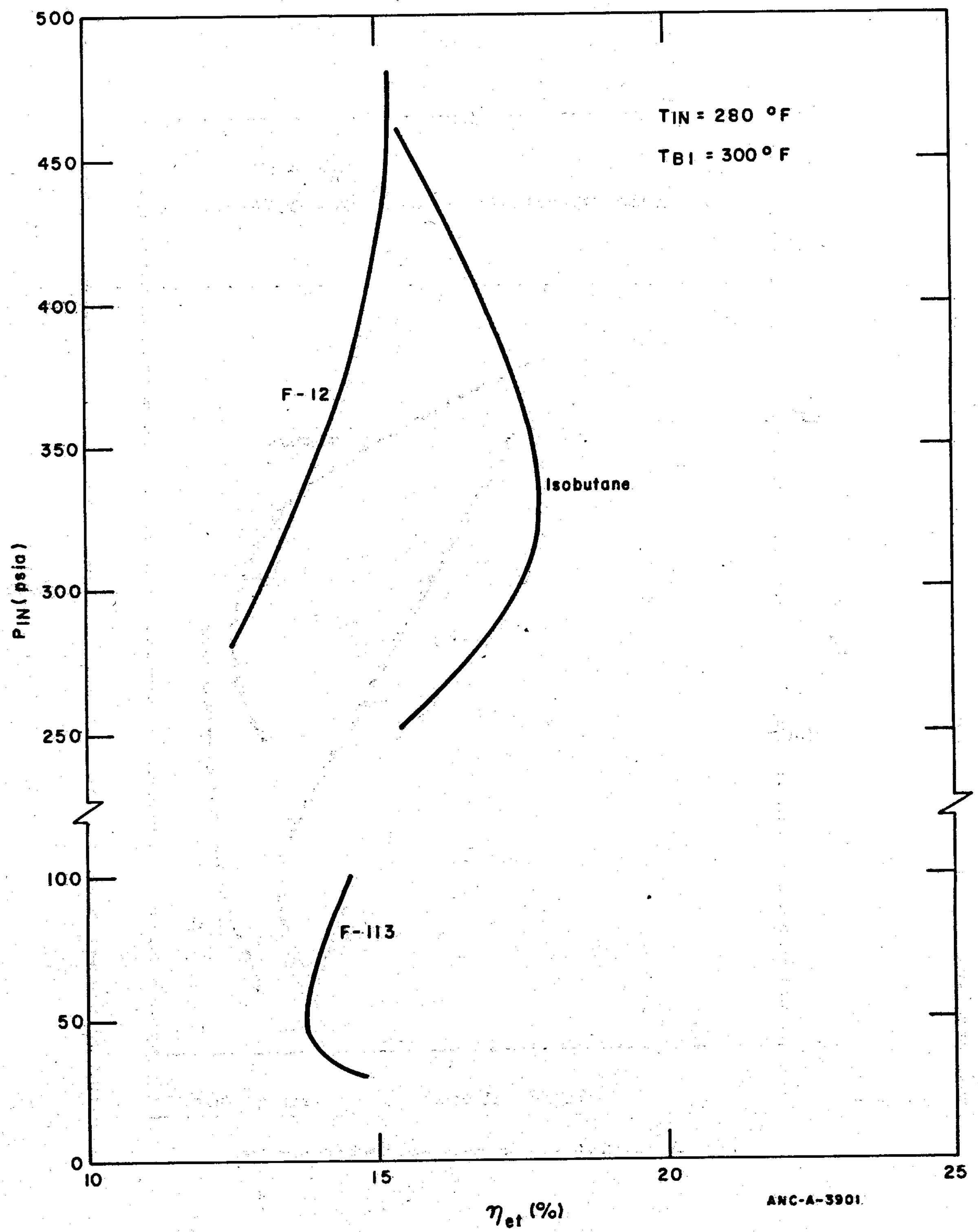

Fig. 11b Superheat cycle thermal efficiency versus turbine inlet pressure.

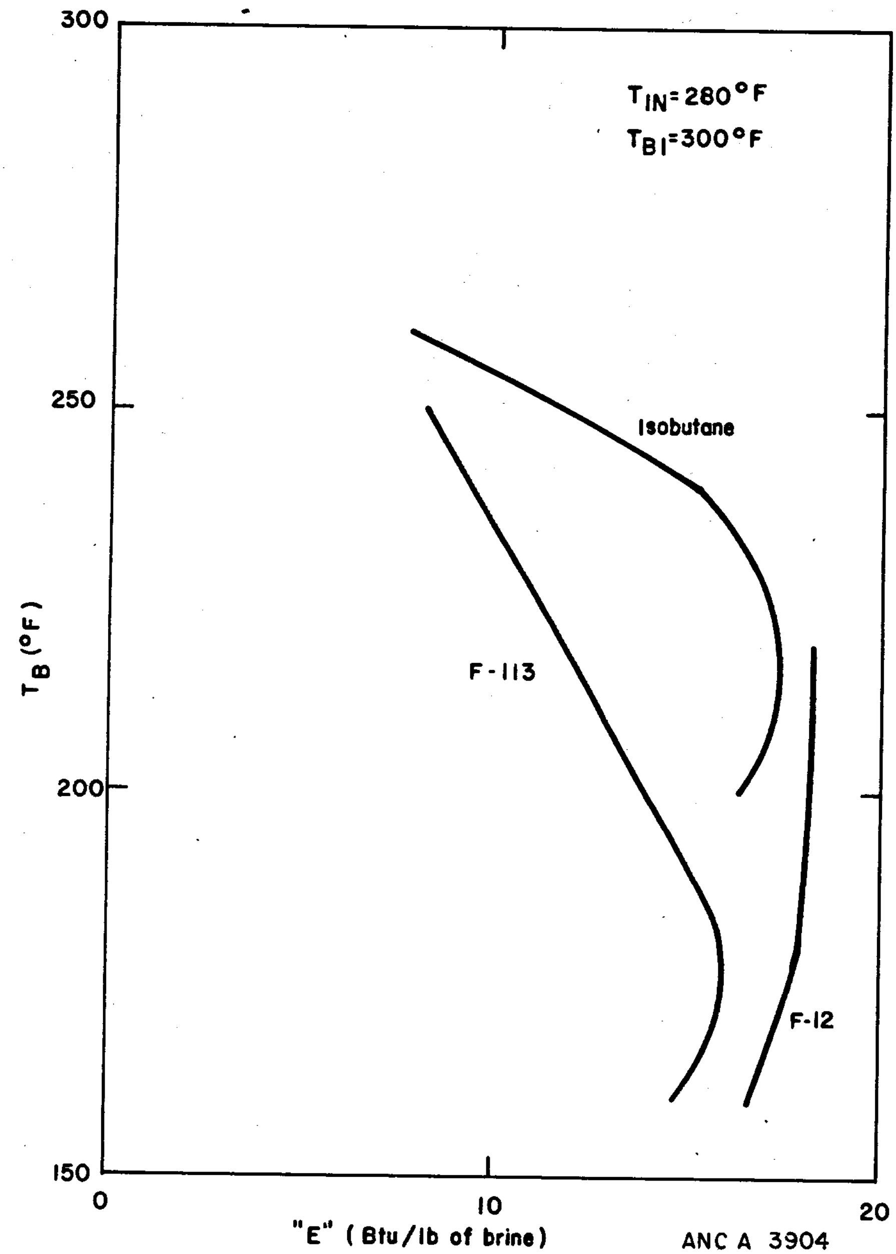

Fig. 12a Superheat cycle effectiveness versus boiler temperature.

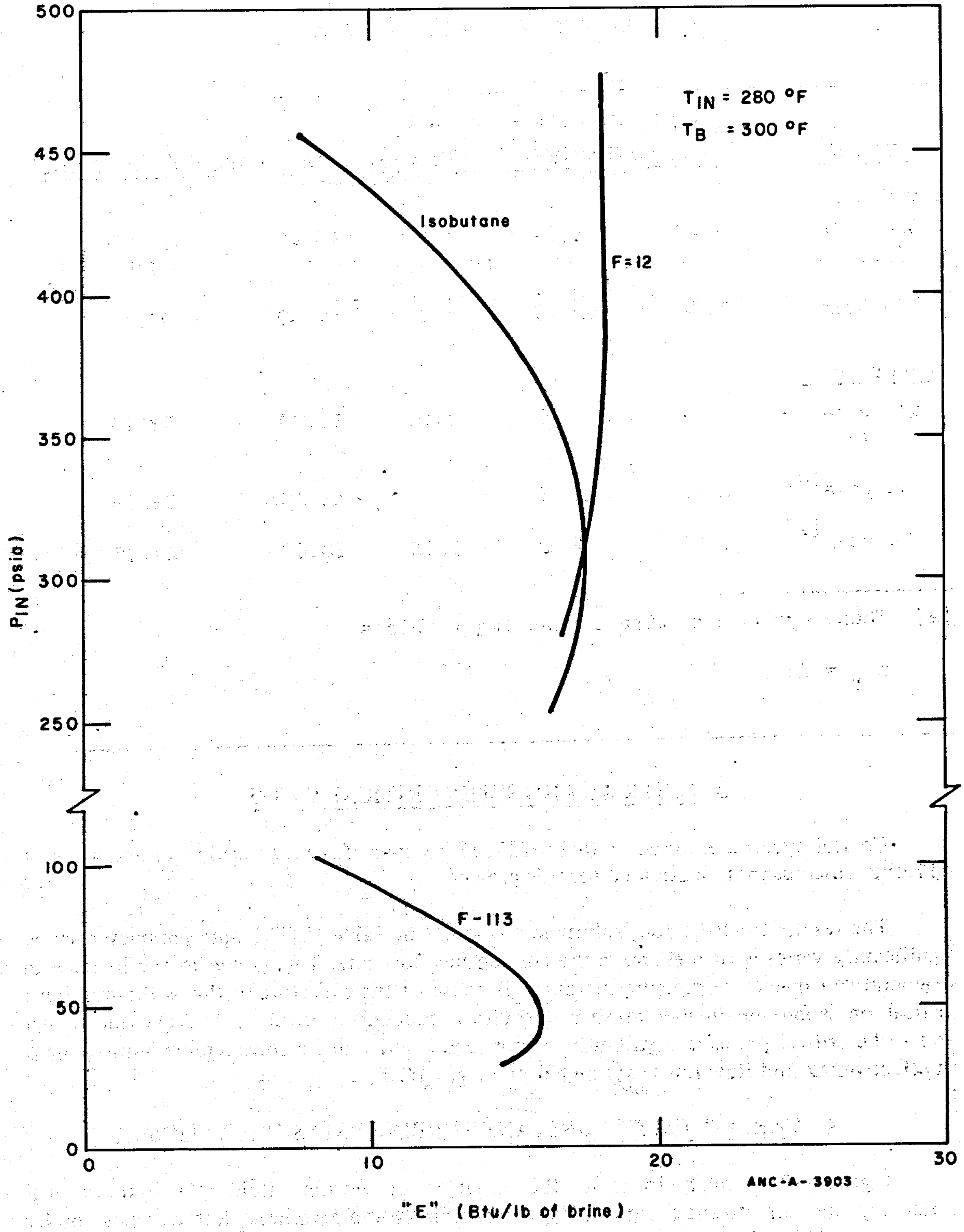

Fig. 12b Superheat cycle effectiveness versus turbine inlet pressure.

TABLE V

CRITICAL AND SUPERCRITICAL CYCLES

Working Fluid	$\dot{w} \times 10^{-6}$ (lb/hr)	$\dot{w}_C \times 10^{-6}$ (lb/hr)	$\dot{w}_B \times 10^{-6}$ (lb/hr)	η_{et} (%)	E (Btu/lb of Brine
Freon-12					
@ 600 psia (cr)	3.43	6.55	1.40	15.42	27.05
@ 700 psia	3.70	15.45	1.39	14.40	27.12
Isobutane					
@ 530 psia (cr)	1.21	7.25	1.14	17.98	33.20
@ 600 psia[a]	2.32	9.73	1.69	11.15	22.35
@ 700 psia[a]	2.57	10.20	1.78	10.63	21.25

[a] These cycles run with condensing turbines

$\eta_t = 75\%$.

3. CRITICAL AND SUPER CRITICAL CASES

Critical cycles are not computed for F-113 because of the high critical temperature of 417.4°F which cannot be attained for this project.

The results for F-12 and isobutane are found in Table V. The only parameter which significantly varies with pressure is the condensing flow rate. This is due to the increase in regenerative capacity as pressure increases. Because of the curvature of the isotherms above critical on isobutane, higher pressures require a condensing turbine. A cycle run at just above the critical pressure is still in the vapor region and shows a considerable improvement in effectiveness and flow rates over higher pressure cycles.

4. VARIATIONS AT CONSTANT TURBINE PRESSURE RATIOS

Figures 13 through 15 show the variation of working fluid flow rate thermal efficiency and effectiveness for varying turbine inlet temperatures, but constant boiler temperature (200°F) and constant turbine inlet pressure. For these curves the parameters in question are a function of temperature only. On the previous curves (Figures 3 through 12), the turbine inlet pressure varies as well as the boiler temperature, but at a different rate.

Figure 13 shows working fluid flow rate versus turbine inlet temperature. As turbine inlet temperature increases, flow rates decrease asymptotically to a minimum value.

Figure 14 shows thermal efficiency curves which increase with temperature. These will approach a maximum as T_{IN} approaches 300°F (neglecting pump work).

Figure 15 illustrates effectiveness. F-113 effectiveness is not very sensitive to turbine inlet temperature changes, since it varies only 1-1/2 Btu/lb of brine in the range shown. It is, however, sensitive to turbine inlet pressure changes as seen in Figure 12b. The curve is straightening out at higher temperatures and will eventually bend back, and the effectiveness will decrease. This is due to pinch point control at higher temperatures.

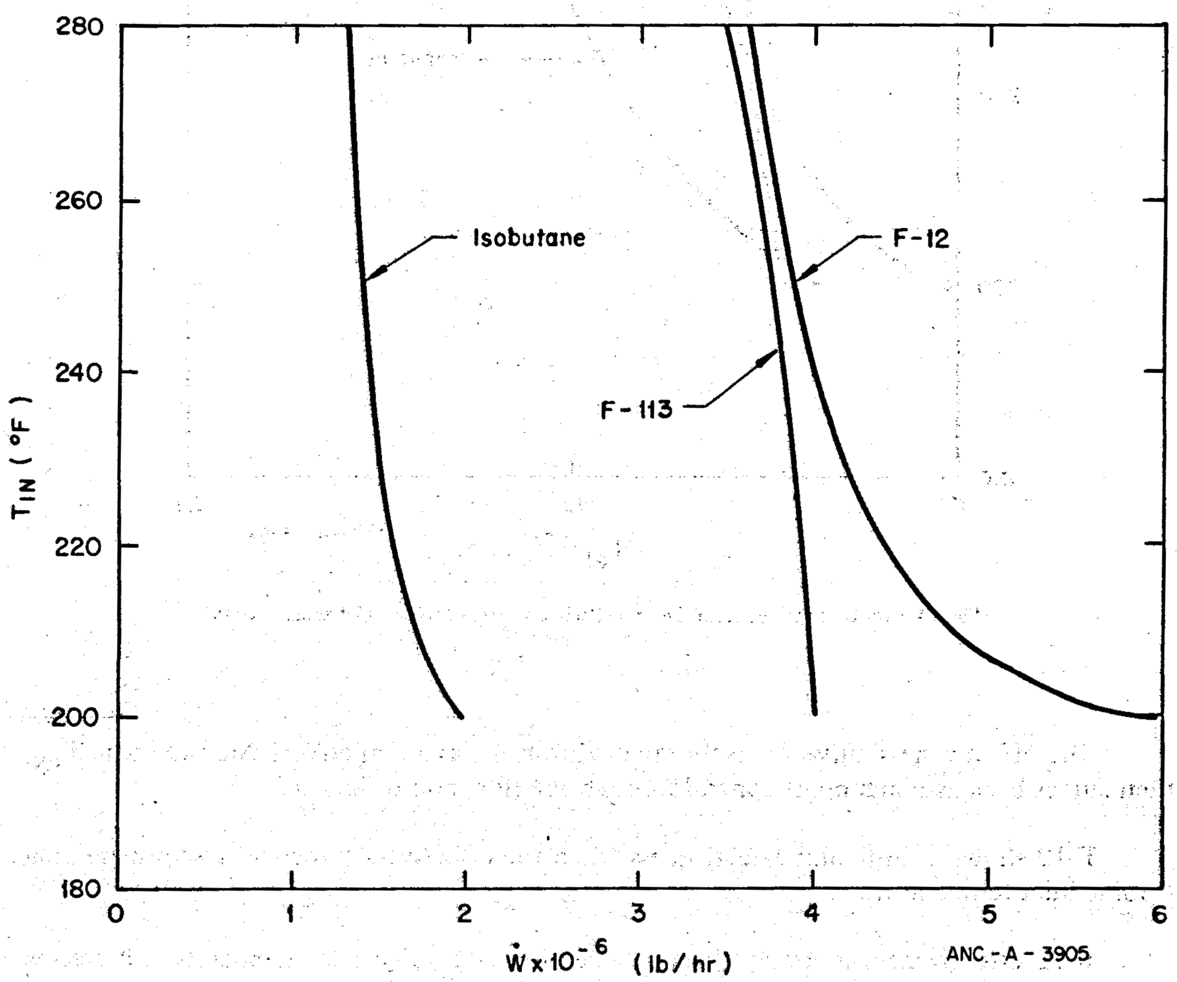

Fig. 13 Constant turbine ratio working fluid flow rate versus turbine inlet temperature.

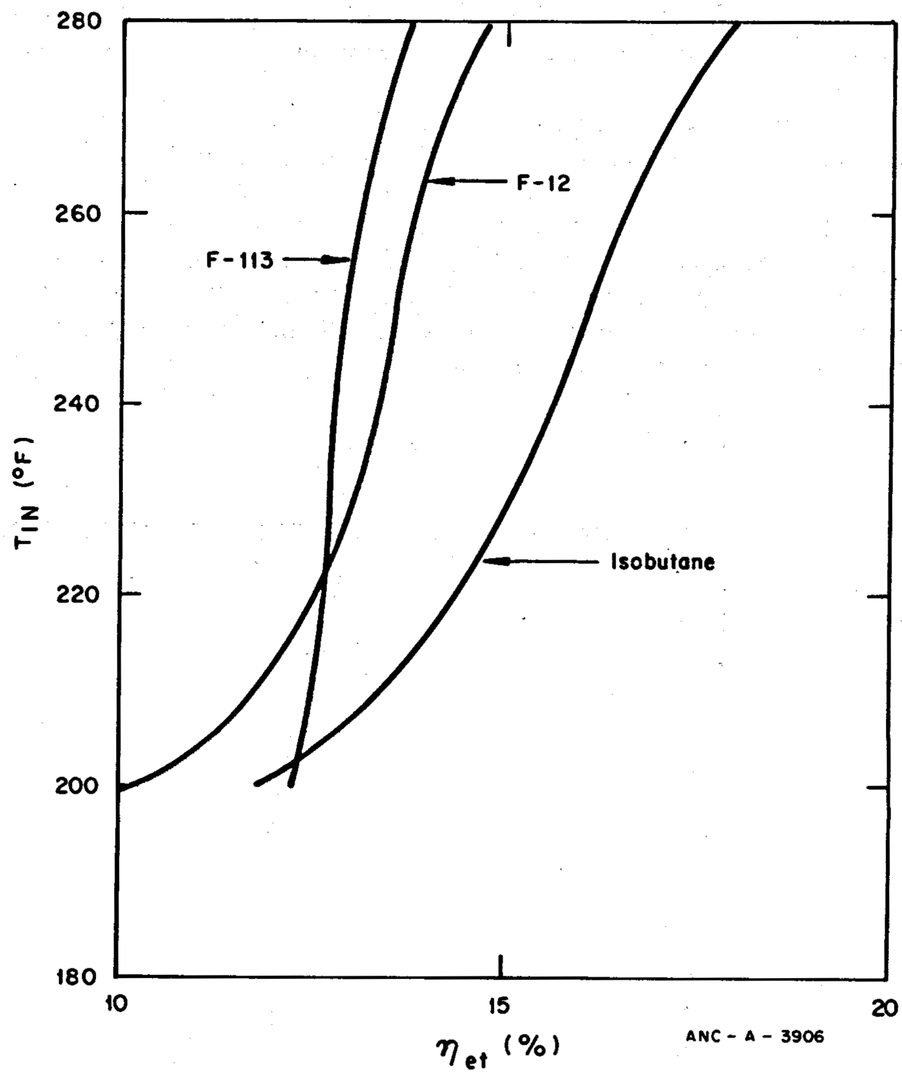

Fig. 14 Constant turbine ratio thermal efficiency versus turbine inlet temperature.

The effectiveness curve for isobutane exhibits a maximum effectiveness at some T_{IN}, then curves back as pinch point control forces brine flow rate to increase.

F-12 shows a minimum effectiveness, then the effectiveness increases with increasing turbine inlet temperature.

F-12 effectiveness is nearly linear, decreasing as turbine inlet temperature increases and pinch point control imposes itself more strongly.

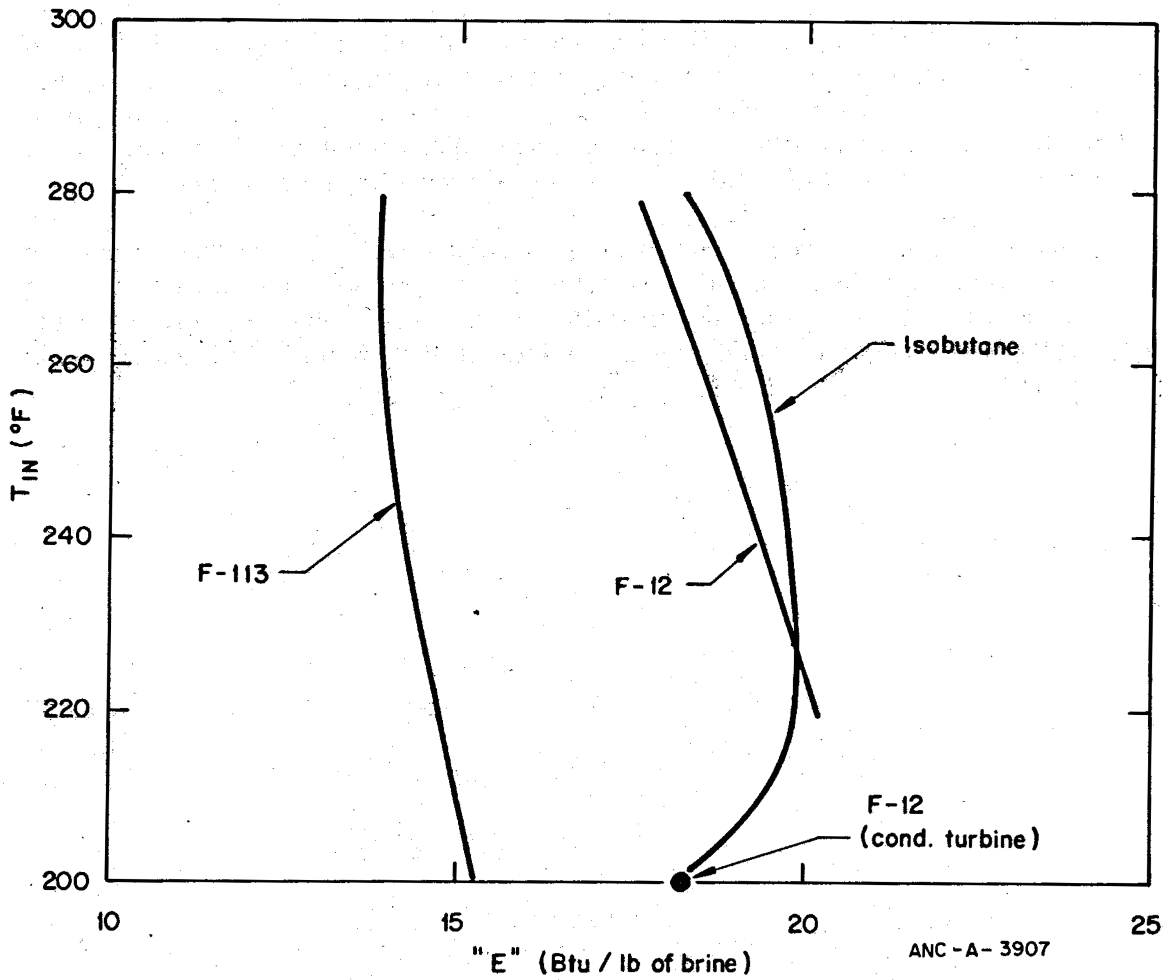

Fig. 15 Constant turbine ratio effectiveness versus turbine inlet temperature.

5. REGENERATION

All curves are based on the use of all the heat available for regeneration. If this heat were not used these would be the effects: Thermal efficiency would decrease due to the increase in external energy input required. Condensing water flow rate would increase, as there would be more vapor at a higher temperature for each case. Brine flow rate will remain the same for 0 superheat and pinch point controlling cases. When the pinch point does not control, brine flow rate will either increase or decrease depending upon the relative magnitudes of Δh_T, CP_L, and Δh_R itself. Effectiveness will vary inversely with brine flow rate. There is no effect on working fluid flow rate or on the amount of superheat (ΔT_S).

VI. CONCLUSIONS AND RECOMMENDATIONS

The criteria listed in Section III indicate that refrigerants as a group would be a good choice for a working fluid. These criteria are based on thermodynamic properties, physical properties, and thermodynamic behavior of the fluid. Care must be taken in the assumptions used for thermodynamic analysis, i.e., some of these fluids exhibit significant variation of C_P, specific heat at constant pressure, and do not behave according to the perfect gas law.

This analysis includes regenerative heating. In some cases it may be that the amount of energy recovered from regeneration is not offset by the size and cost of the heat exchanger required. Time does not permit further work in this analysis but a study of the cycles without regeneration would present an idea of the proportionate gain in using regeneration.

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APPENDIX

DERIVATION OF EQUATIONS

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DERIVATION OF EQUATIONS

1. TURBINE WORK

$$h = u + pv$$

$$dh = du + pdv + vdp$$

$$Tds = du + pdv$$

$$\therefore dh = Tds + vdp.$$

For $ds = 0$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{K-1}{K}}$$

$$dh = vdp$$

$$pv = RT \text{ or } v = \frac{RT}{p}$$

$$Cp = \left(\frac{dh}{dt}\right)_p$$

$$dh = RT\,\frac{dp}{p}$$

$$Cp\,\frac{dT}{T} = R\,\frac{dp}{p}$$

$$\frac{dT}{T} = \frac{R}{Cp}\,\frac{dp}{p}$$

$$\ln T\Big[_1^2 = \frac{R}{Cp}\ln p\Big]_1^2 + \cancel{C}^{\,0}$$

$$\ln T_2 - \ln T_1 = \frac{R}{Cp}\left[\ln p_2 - \ln p_1\right].$$

$$\ln\frac{T_2}{T_1} = \frac{R}{Cp}\ln\frac{p_2}{p_1}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{R}{Cp}}$$

$$R = Cp - Cv$$

$$\frac{R}{Cp} = 1 - \frac{Cv}{Cp} = 1 - \frac{1}{k} = \frac{k-1}{k}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$$

$$\frac{T_2}{T_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}}$$

$T_1 = T_{IN}; \quad T_2 = T_{OUT}$

at ds = 0, work = $\dot{w}Cp(T_{IN} - T_{OUT})$

$$T_{IN} = T_1; \; T_{OUT} = T_1 \left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}}$$

$$W = \dot{w}\; Cp \left[T_1 - T_1 \left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}} \right]$$

$$= \dot{w}\; Cp\; T_{IN} - \left[1 - \left(\frac{p_{IN}}{p_{OUT}}\right)^{\frac{1-k}{k}} \right] .$$

But the turbine has an efficiency of η_t.

$$T_{OUT_{actual}} = \eta_t \; T_{OUT_{ideal}}$$

$$W = \dot{w}\; Cp\; T_{IN}\; \eta_t \left[1 - \left(\frac{p_{IN}}{p_{OUT}}\right)^{\frac{1-k}{k}} \right] .$$

2. PUMP WORK

$dh)_s = vdp$

Assume incompressible liquid $\therefore$ dp = Δp

p_{in} to pump $- \; p_{out}$ from pump $= \Delta p$

ΔP_{PO} = pressure drop across boiler (positive)

ΔP_{PI} = pressure drop across condenser (positive)

p_{IN} pump $= p_{OUT}$ turbine $- \Delta P_{PI}$

p_{OUT} pump $= p_{IN}$ turbine $+ \Delta p_{PO}$

$\therefore \; dh)_s = v \; (p_{IN} + \Delta p_{PO} - p_{OUT} + \Delta P_{PI})$

$dh)_s = v \; (p_{IN} - p_{OUT} + \Delta p_{PO} + \Delta p_{PI})$

$dh)_s = C_p \; dT)_s$

$C_{p_L} \; \Delta T = v(\Sigma p)$.

For pump efficiency $= \eta_p$, $\Delta T_{actual} = \eta \; \Delta \; T_{ideal}$

$$\therefore \; \Delta T = \frac{v}{C_{PL} \; \eta_p} \; (\Sigma p).$$

$$\text{Work} = \dot{w} \; cp \; \Delta T$$

$$= \dot{w} \frac{v}{\eta_p} (\Sigma p)(CF) \qquad CF = \frac{144}{778} \text{ (a conversion factor for units).}$$

$$\text{Net work} = \dot{w} \left[C_p \; T_{IN} \eta_t \; (1 - (\frac{P_{IN}}{P_{OUT}})^{\frac{1-k}{k}}) - \frac{vCF}{\eta_p} \; (\Sigma p) \right] .$$

We need W_N (Net work) $= 3.79 \times 10^7 \; \frac{Btu}{hr}$

$$\therefore \; \dot{w} = \frac{W_N}{Cp \; T_{IN} \; \eta_t \left[1 - (\frac{P_{IN}}{P_{OUT}})^{\frac{1-k}{k}} \right] - \frac{vCF}{\eta_p} \; (\Sigma P)} .$$

3. REGENERATOR

$\Delta Tregen = T_{OUT} - T_{COND}$

$Qregen = \dot{w} \; c_{p_G} \; (\Delta Treg_G) = \dot{w} \; C_{PL} \; (\Delta Treg_L)$.

(gas side) (liquid side)

On gas side, $\Delta Treg_G = T_{OUT} - T_{COND}$.

On liquid side, $\Delta Treg_L = T_{reg_{out}} - T_{pump_{out}}$

so: $Cp_G \; (T_{OUT} - T_{COND}) = C_{PL} \; (T_{reg_{out}} - T_{pump_{out}})$.

Assume incompressible liquid, so $T_{COND} \simeq T_{pump_{out}}$

$$\frac{C_{PG}}{C_{PL}} (T_{OUT}) - \frac{C_{PG}}{C_{PL}} (T_{COND}) = Treg_{out} - T_{COND}$$

$$\frac{C_{PG}}{C_{PL}} (T_{OUT}) - T_{reg_{OUT}} = \frac{C_{PG}}{C_{PL}} (T_{COND}) - T_{COND}$$

$$T_{reg_{out}} = (1 - \frac{C_{PG}}{C_{PL}}) T_{COND} + \frac{C_{PG}}{C_{PL}} (T_{OUT}) .$$

But $T_{OUT} = \eta_t \; T_{IN} \; (\frac{P_{IN}}{P_{OUT}})^{\frac{1-k}{k}}$.

So: $T_{reg_{out}} = (1 - \frac{C_{PG}}{C_{PL}}) T_{COND} + \frac{C_{PG}}{C_{PL}} \eta_t \; T_{IN} \; (\frac{P_{IN}}{P_{OUT}})^{\frac{1-k}{k}}$

$\Delta h_R = C_{PL} \; \Delta T_R$ (available to cycle)

$T_{reg\;IN} = T_{PO} \simeq T_{COND}$.

Then:

$$\Delta h_R = C_{PL} \left\{ - T_{COND} + \frac{CpL - Cp_G}{Cp_L} (T_{COND}) + \frac{C_{PG}}{C_{PL}} \left[\eta_t \; T_{IN} \; (\frac{P_{IN}}{P_{OUT}})^{\frac{1-k}{k}} \right] \right\}$$

$$= C_{P_L} T_{COND} - C_{PL} T_{COND} - C_{PG} T_{COND} + C_{PG} \left[\eta_t \; T_{IN} \; (\frac{P_{IN}}{P_{OUT}})^{\frac{1-k}{k}} \right]$$

$$\Delta h_R = Cp_G \left\{ \left[\eta_T \; T_{IN} \; (\frac{P_{IN}}{P_{OUT}})^{\frac{1-k}{k}} \right] - T_{COND} \right\} .$$

For convenience, since the term $C_{PG} \; T_{IN} \; \eta_t \; (1 - \frac{P_{IN}}{P_{OUT}})^{\frac{1-k}{k}}$ is already used, and $= Cp_G \; (T_{IN} - T_{OUT})$, then

$$T_{OUT} = - \frac{C_{P_G} T_{IN} \eta_t \left(1 - \frac{P_{IN}}{P_{OUT}}\right)^{\frac{1-k}{k}}}{C_{P_G}} + T_{IN}$$

so: $$\Delta h_R = C_{P_G} \left[- T_{in} \eta_t \left(1 - \frac{P_{IN}}{P_{OUT}}\right)^{\frac{1-k}{k}} + T_{IN} - T_{COND} \right]$$

$$\Delta h_R = C_{P_G} \left[T_{IN} - T_{COND} - T_{IN} \eta_t \left(1 - \left(\frac{P_{IN}}{P_{OUT}}\right)^{\frac{1-k}{k}}\right) \right]$$

4. <u>Δh_{TOTAL} (input)</u>

$$\Delta h_T = \Delta h_{Boil} + \underset{w/o\ reg}{\Delta h_{Heater}} + \Delta h_{super} - \Delta h_{reg}$$

$$\Delta T_{Heater} = T_{Boil} - T_{reg_{out}}$$

$$T_{reg_{out}} = \left(1 - \frac{Cp_G}{Cp_L}\right) T_{COND} + \frac{Cp_G}{Cp_L} \left[\eta_t T_{IN} \left(\frac{P_{IN}}{P_{OUT}}\right)^{\frac{1-k}{k}} \right]$$

$$\Delta T_H = T_B - \left(1 - \frac{C_{pG}}{C_L}\right) T_{COND} - \frac{C_{pG}}{C_{pL}} \left[\eta_t T_{IN} \left(\frac{P_{IN}}{P_{OUT}}\right)^{\frac{1-k}{k}} \right]$$

$$\Delta h_H = C_{PL} \Delta T_H$$

$$\Delta h_H = C_{pL} \left[T_B - \left(1 - \frac{C_{pG}}{C_{pL}}\right) T_{COND} - \frac{C_{pG}}{C_{pL}} \left\{ \eta_t T_{IN} \left(\frac{P_{IN}}{P_{OUT}}\right)^{\frac{1-k}{k}} \right\} \right]$$

$$\Delta T_{super} = T_{IN} - T_B$$

$$\Delta h_{super} = C_{pG} (T_{IN} - T_B).$$

Now, for no regen, $\Delta h_H + C_{pL} (T_B - T_{COND})$. For regen, $\Delta h_H = Cp_L (T_B - T_{COND}) - \Delta h_{reg}$.

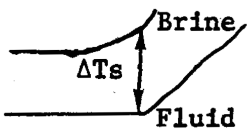
So: $\Delta h_T = \Delta h_B + C_{pL} (T_B - T_{COND}) - \Delta h_R + C_{pG} (T_{IN} - T_B)$.

5. THERMAL EFFICIENCY

$$\eta_{et} + \frac{\text{Work Out (Net)}}{\text{Heat added X } \dot{w}}$$

$$\eta_{et} = \frac{W_N}{\Delta h_T \; \dot{w}}$$

$\underline{\Delta T_s}$



(See Section IV-2 for an explanation of pinch point)

$$\dot{w}_{Boiler} = \frac{\overbrace{\dot{w} \; \Delta h_B}^{\text{(heat required by boiler)}}}{\underbrace{\underset{1}{CP_{H_2O}} \; (\Delta T_s - \Delta T_p)}_{\text{(heat available from brine)}}}$$

$$\dot{w}_{Super} = \frac{\overbrace{\dot{w} \; Cp_G \; (T_{IN} - T_B)}^{\text{(heat required by superheater)}}}{\underbrace{\underset{1}{Cp_{H_2O}} \; (T_{BI} - T_B - \Delta T_s)}_{\text{(heat available from brine)}}}$$

$\dot{w}_{Boiler}$ must $= \dot{w}_{super}$.

Then

$$\frac{(\dot{w} \; \Delta h_B)}{\Delta T_s - \Delta T_p} = \frac{\dot{w} \; C_{PG} \; (T_{IN} - T_B)}{(T_{BI} - T_B - \Delta T_s)}$$

$$\Delta h_B \; (T_{BI} - T_B - \Delta T_s) = C_{PG} \; (T_{IN} - T_B)(\Delta T_s - \Delta T_p)$$

$$\Delta h_B \; (T_{BI} - T_B) - \Delta h_B \; (\Delta T_S) = C_{pG} \; (T_{IN} - T_B)(\Delta T_s) - C_{pG} \; (T_{IN} - T_B)(\Delta T_p)$$

$$\Delta T_S \left[+ \Delta h_B + C_{pG} \; (T_{IN} - T_B) \right] = + \Delta h_B \; (T_{BI} - T_B) + C_{pG} \; (T_{IN} - T_B) \; \Delta T_p$$

$$\Delta T_S = \frac{\Delta h_B \; (T_{BI} - T_B) + C_{pG} \; \Delta T_p \; (T_{IN} - T_B)}{\Delta h_B + C_{pG} \; (T_{IN} - T_B)} \; .$$

6. CONDENSING H_2O

amount of fluid to be condensed = $\dot{w}_f$

heat needed to be extracted = $\Delta h_{COND} + \Delta h_R$

heat available from cooling H_2O = $C_{p\,H_2O}\,\Delta T_{COND}$

$$\therefore\ \dot{w}_c = \frac{\dot{w}\,(\Delta h_{COND} + \Delta h_R)}{\Delta T_{COND}}\ .$$

6.1 Brine Outlet Temperature and Brine Flow Rate

(1) 0 Superheat Case, $\Delta T_S \geq T_{BI} - T_B$:

First we must see if the boiler flow rate or the heater flow rate controls. The boiler flow rate:

$$Q_{Brine} = \dot{w}_B \left[(T_B + \Delta T_S) - (T_B + \Delta T_P)\right] C_{P\,\cancel{H_2O}}^{\ 1}$$

$$= \dot{w}_B\,(\Delta T_S - \Delta T_P)$$

$$Q_{Fluid} = \dot{w}\,\Delta h_B$$

$$\dot{w}_B\,(\Delta T_S - \Delta T_P) = \dot{w}\,\Delta h_B$$

$$\dot{w}_B = \frac{\dot{w}\,\Delta h_B}{\Delta T_S - \Delta T_P}\ .$$

The heater flow rate:

$$Q_{Brine} = \dot{w}_B C p_{\cancel{H_2O}}^{\ 1}\,\Delta T_{Brine}$$

$$\Delta T_{Brine} = (T_B + \Delta T_P) - T_{BO}$$

$$Q_{Brine} = \dot{w}_B\,(T_B + \Delta T_P - T_{BO})$$

$$Q_{Fluid} = \dot{w}\,(T_B - Treg_{out})$$

$$\dot{w}_B(T_B + \Delta T_P - T_{BO}) = \dot{w}(T_B - Treg_{out})\,C_{PL}$$

$$Treg_{out} = T_{COND} + \Delta Treg = T_{COND} + \frac{\Delta h_R}{C_{PL}}$$

then,

$$\dot{w}_B (T_B + \Delta T_P - T_{BO}) = \dot{w} \left[C_{PL} \; (T_B - T_{COND}) - \Delta h_R \right]$$

or

$$\dot{w}_B = \frac{\dot{w} \left[C_{PL} (T_B - T_{COND}) - \Delta h_R \right]}{(T_B + \Delta T_P - T_{BO})} \; .$$

If the boiler flow controls,

$$\frac{\dot{w} \; \Delta h_B}{\Delta T_S - \Delta T_P} > \frac{\dot{w} \left[C_{PL} \; (T_B - T_{COND}) - \Delta h_R \right]}{(T_B + \Delta T_P - T_{BO})} \; .$$

For the best boiler, $\Delta T_P = 0$.

Then

$$\frac{\Delta h_B}{\Delta T_S} > \frac{C_{PL} \; (T_B - T_{COND}) - \Delta h_R}{T_B - T_{BO}} \; .$$

Now, for the best boiler, $\dot{w}_B$ is minimum, and

$$\dot{w}_{B_{Boil_{Min}}} = \frac{\dot{w} \; \Delta h_B}{\Delta T_S} \; .$$

At this flow rate, max Q_{Heater} is

$$Q_{H_{Max}} = \dot{w}_{B_{Boil_{Min}}} \; \cancelto{1}{C_{P_{H_2O}}} \; (T_B + \Delta T_P - T_{BO}) \, .$$

But at the same time, the heat accepted by the fluid is

$$Q_H = \dot{w} \; C_{PL} \; (T_B - T_{COND} - \frac{\Delta h_R}{Cp_L})$$

$$= \dot{w} \left[Cp_L \; (T_B - T_{COND}) - \Delta h_R \right]$$

$\dfrac{Q_H}{Q_{H_{Max}}} \leq 1$ (by definition), therefore

$$\frac{\dot{w}_{B_{Boil_{Min}}} (T_B + \Delta T_p - T_{BO})}{\dot{w}\left[C_{PL} (T_B - T_{COND}) - \Delta h_R\right]} \leq 1 .$$

But $\dot{w}_{B_{Boil_{Min}}} = \dfrac{\dot{w}\ \Delta h_B}{\Delta T_S}$

then

$$\frac{\dot{w}\ \Delta\ h_B\ (T_B + \Delta T_p - T_{BO})}{\dot{w}\ \Delta T_S \left[Cp_L\ (T_B - T_{COND}) - \Delta h_R\right]} \leq 1$$

or

$$\frac{T_B + \Delta Tp - T_{BO}}{Cp_L\ (T_B - T_{COND}) - \Delta h_R} \leq \frac{\Delta\ T_S}{\Delta h_B}$$

and

$$\frac{\Delta h_B}{\Delta T_S} > \frac{C_{PL}\ (T_B - T_{COND}) - \Delta h_R}{T_B + \Delta T_p - T_{BO}} .$$

which is what we had before, so the boiler flow is greater and controls the brine flow rate.

$$\therefore\ \dot{w}_B = \frac{\dot{w}\Delta h_B}{\Delta T_S - \Delta\ T_p} .$$

For the heater, at the $\dot{w}_B$ above,

$$Q_{Brine} = \dot{w}_B\ \cancelto{1}{C_{P_{H_2O}}}\ (\Delta T_{Brine})$$

$$\Delta T_{Brine} = (T_B + \Delta T_p - T_{BO})$$

$$\therefore\ Q_{Brine} = \dot{w}_B\ (T_B + \Delta T_p - T_{BO})$$

$$Q_{Fluid} = \dot{w}\left[C_{PL}\ (T_B - T_{COND}) - \Delta h_R\right]$$

$$\dot{w}_B\ (T_B + \Delta T_p - T_{BO}) = \dot{w}\left[C_{P_L}\ (T_B - T_{COND}) - \Delta h_R\right]$$

$$\dot{w}_B = \frac{\dot{w}\ \Delta h_B}{\Delta T_S - \Delta T_p}$$

$$\therefore \frac{\dot{w}\ \Delta h_B\ (T_B + \Delta T_p - T_{BO})}{\Delta T_S - \Delta T_p} = \dot{w}\left[C_{PL}\ (T_B - T_{COND}) - \Delta h_R\right]$$

$$T_B + \Delta T_p - T_{BO} = \frac{\left[C_{P_L}\ (T_B - T_{COND}) - \Delta h_R\right]\left[\Delta T_S - \Delta T_p\right]}{\Delta h_B}$$

or

$$T_{BO} = T_B + \Delta T_p - \frac{\left[C_{P_L}\ (T_B - T_{COND}) - \Delta h_R\right]\left[\Delta T_S - \Delta T_p\right]}{\Delta h_B}\ .$$

(2) Superheat Case:

First check if ΔT_P controls (see Section IV-2 for an explanation of pinch point control)

ΔT_P must $\leq T_{BI} - T_B$

also T_{BO} must $\geq Treg_{out}$

$$Treg_{out} = (1 - \frac{C_{PG}}{C_{PL}})\ T_{COND} + \frac{C_{PG}}{C_{PL}}\ (T_{out})\ .$$

To check if ΔT_P controls, calculate T_{BO} as if it did. To do this, we must calculate $\dot{w}_B$ for ΔT_P control.

$$Q_{Boiler} = \dot{w}\Delta h_B\ \text{(req'd)}$$

then, corresponding $Q_B = \dot{w}_B \cancelto{1}{C_{P_{H_2O}}} \left[T_{BI} - (T_B + \Delta T_P)\right]$; $T_B + \Delta T_P = T_{BO}$

$$\dot{w}_B = \frac{\dot{w}\ \Delta\ h_B}{T_{BI} - T_B - \Delta T_P}$$

$$Q_{Super} \doteq \dot{w}Cp(T_{IN} - T_B)\ \text{(req'd)}$$

corr, $Q_B = \dot{w}_B \cancelto{1}{Cp_{H_2O}} \left(T_{BI} - (T_B + \Delta T_S)\right)$; $T_B + \Delta T_S = T_{BO}$

$$\dot{w}_B = \frac{\dot{w}\, C_{P_G}\, (T_{IN} - T_B)}{T_{BI} - T_B - \Delta T_S}$$

$$Q_{heater} = \dot{w}\, Cp_L (T_B - T_{reg_{out}}) \text{ (req'd)}$$

$$\text{corr, } Q_B = \dot{w}_B\, \cancel{C_{P_{H_2O}}}^{\,1} \left[(T_B + \Delta T_P) - (T_{reg_{out}} + \Delta T_2) \right]$$

where $\Delta T_2 = T_{BO} - T_{reg_{out}}$ (HX parameter)

$$\text{and } \dot{w}_B = \frac{\dot{w}\, C_{P_L}\, (T_B - T_{reg_{out}})}{T_B + \Delta T_P - T_{reg_{out}} + \Delta T_2}$$

$\dot{w}$ for heater or boiler or superheater will govern, whichever is greatest.

First see if $\dot{w}_{B_{Heater}}$ > or < $\dot{w}_{B_{Boiler}}$.

If boiler flow is limiting,

$$\frac{\dot{w}\, \Delta h_B}{T_{BI} - T_B - \Delta T_P} > \frac{\dot{w}\, C_{PL}\, (T_B - T_{reg_{out}})}{T_B + \Delta T_P - T_{reg_{out}} - \Delta T_2} .$$

If we build the best boiler and the best heater, $\Delta T_P = 0$ and $\Delta T_2 = 0$

$$\text{then: } \frac{\Delta h_B}{T_{BI} - T_B} > \frac{Cp_L\, (T_B - T_{reg_{out}})}{(T_B - T_{reg\ out})}$$

$$\text{or } \frac{\Delta h_B}{Cp_L\, (T_{BI} - T_B)} > 1 .$$

Now, when we have the best boiler and heater, we can find the minimum

$\dot{w}_{B_{Boiler}}$

$$\text{so } \dot{w}_{B_{Boil\ Min}} = \frac{\dot{w}\ \Delta h_B}{T_{BI} - T_B} .$$

For this flow, the max heat to the heater is $Q_{H_{Max}} = \dot{w}_{B_{Boil\ Min}} (T_B - T_{reg_{out}})$

$$\text{or } T_B - T_{reg\ out} = \frac{Q_{H_{Max}}}{\dot{w}_{B_{Boil\ Min}}} .$$

But, on the working fluid side of the heater, $T_B - T_{reg_{out}} = \Delta T$ through

the heater and $Q_H = \dot{w}\ C_{P_L} (T_B - T_{reg_{out}})$.

$$\text{Now, } \frac{Q_H}{Q_{H_{max}}} \leq 1$$

$$\text{or } \frac{\dot{w}\ C_{P_L} (T_B - T_{reg_{out}})}{\dot{w}_{B_{Boil_{Min}}} (T_B - T_{reg_{out}})} \leq 1 .$$

$$\text{But } \dot{w}_{B_{Boil_{Min}}} = \frac{\dot{w}\ \Delta h_B}{T_{BI} - T_B}$$

$$\therefore \frac{\dot{w}\ C_{P_L} (T_{BI} - T_B)}{\dot{w}\ \Delta h_B} \leq 1$$

or $$\frac{\Delta h_B}{C_{P_L} \; (T_{BI} - T_B)} \; > \; 1$$

which is what we had before and therefore the boiler controls the flow.

Now, check if $\dot{w}_{B_{Boiler}}$ > or < $\dot{w}_{B_{super}}$.

If the boiler controls the flow,

$$\frac{\dot{w} \; \Delta h_B}{T_{BI} - T_B - \Delta T_P} \quad > \quad \frac{\dot{w} \quad C_{PG} \; (T_{IN} - T_B)}{T_{BI} - T_B - \Delta T_S} \; .$$

For best boiler, $\Delta T_P = 0$, and

$$\frac{\Delta \; h_B}{T_{BI} - T_B} \quad > \quad \frac{C_{P_G} \;\; (T_{IN} - T_B)}{T_{BI} - T_B - \Delta T_S} \; .$$

Then, for minimum boiler flow rate,

$$\dot{w}_{B_{Boil_{Min}}} \quad = \quad \frac{\dot{w} \; \Delta h_B}{T_{BI} - T_B} \; .$$

For this flow rate, maximum heat flow into the superheater is

$$Q_{S_{Max}} = \dot{w}_{B_{Boil_{Min}}} \; \cancelto{1}{C_{P_{H_2O}}} \left[T_{BI} - (T_B + \Delta T_S) \right]$$

or

$$\dot{w}_{Boil_{Min}} \; = \; \frac{Q_{S_{Max}}}{T_{BI} - T_B - \Delta T_S} \; .$$

At the same time, the working fluid side of the superheater accepts

$$Q_S = \dot{w}\, C_{P_G}\, (T_{IN} - T_B)$$

$$\frac{Q_S}{Q_{S_{Max}}} \leq 1$$

then

$$\frac{\dot{w}\, C_{P_G}\, (T_{IN} - T_B)}{\dot{w}_{B_{Boil_{Min}}}\, (T_{BI} - T_B - \Delta T_S)} \leq 1 .$$

But

$$\dot{w}_{B_{Boil_{Min}}} = \frac{\dot{w}\, \Delta h_B}{T_{BI} - T_B} .$$

So,

$$\frac{\dot{w}\, C_{P_G}\, (T_{IN} - T_B)(T_{BI} - T_B)}{\dot{w}\, \Delta h_B\, (T_{BI} - T_B - \Delta T_S)} \leq 1.$$

By inspection, this is not true, nor does it correspond to the previous inequality $\therefore$ the boiler does not control the flow, the superheater controls, and the equation to use is:

$$\dot{w}_B = \frac{\dot{w}\, C_{P_G}\, (T_{IN} - T_B)}{T_{BI} - T_B - \Delta T_S} .$$

For the heater, on the brine side,

$$Q_H = \dot{w}_B\, (T_B + \Delta T_p - T_{BO})\, Cp_{\cancelto{1}{H_2O}} .$$

For the working fluid,

$$Q_H = \dot{w}\, C_{P_L}\, (T_B - T_{COND})$$

$$\therefore \dot{w}_B\ (T_B + \Delta T_p - T_{BO}) = \dot{w}\ C_{P_L}\ (T_B - T_{COND}) - \Delta h_R .$$

But

$$\dot{w}_B = \frac{\dot{w}\ C_{P_G}(T_{IN} - T_B)}{T_{BI} - T_B - \Delta T_S} .$$

$$T_{BO} = T_B + \Delta T_P - \frac{\left[C_{P_L}\ (T_B - T_{COND}) - \Delta h_R\right]\ \left[T_{BI} - T_B - \Delta T_S\right]}{C_{P_G}\ (T_{IN} - T_B)} .$$

This applies to cases when the pinch point controls. If $T_{BO} \leq T_{COND} + \frac{\Delta h_R}{C_{P_L}}$ the pinch point does not control, and

$$T_{BO} = T_{COND} + \frac{\Delta h_R}{C_{P_L}} + \Delta T_2$$

then Qin to the fluid = $\dot{w}\ \Delta hT$

and Qout of the Brine = $(T_{BI} - T_{BO})\ \dot{w}_B\ \cancelto{1}{C_{P_{H_2O}}}$

so,

$$\dot{w}\ \Delta\ hT = \dot{w}_B\ (T_{BI} - T_{BO})$$

or

$$\dot{w}_B = \frac{\dot{w}\ \Delta h_T}{T_{BI} - T_{BO}} .$$

Where

$$T_{BO} - T_{COND} + \frac{\Delta h_R}{C_{P_L}} + \Delta T_2$$

or

$$\dot{w}_B = \frac{\dot{w}\ \Delta h_T}{T_{BI} - T_{COND} - \frac{\Delta h_R}{C_{P_L}} - \Delta T_2} .$$

The supercritical case is classed as pinch point not controlling since there is no boiler.

7. EFFECTIVENESS

The effectiveness is a measure of the amount of energy extracted from the brine,

$$E = \frac{W_N}{\dot{w}_B} \quad \frac{\text{Btu}}{\text{lb}} \text{ of Brine.}$$